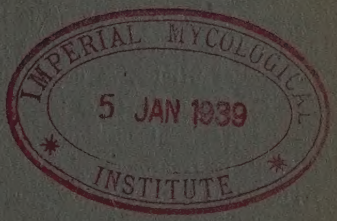
COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

1938



VOLUME XX

№ 4

ÉDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
MOSCOU

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5 Août

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(DOKLADY)

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NOUVELLE SERIE

6-me année

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1938

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En composition 26/VII 1938. En impression 9/IX. 1938. Nombre de feuilles d'impresson 6¼. Nombre de caractères par feille d'impression 65.000. Pages 233—336. Format du papier 72×110 cm. Tirage 1.000. Glavlit B 50403. Numéro d'édition 795. Numéro typographique 1124

Imprimerie № 16. Moscou. Trekhproudnjj péréoulok, 9.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

MATHÉMATIQUES

SUR QUELQUES PROPRIÉTÉS DES COURANTS DISCONTINUS D'UN FLUIDE

Par M. LAVRENTIEFF

(Présenté par I. M. Vinogradow, de l'Académie, le 27. V. 1938)

Le but de la présente Note est de compléter les résultats exposés dans ma Note «Sur la théorie des sillages» (C. R. Acad. Sci. URSS, XVIII, № 4—5, 1938).

1. Soit γ une courbe simple jordanienne fermée, partiellement unie (stückenweise glatt), située dans le plan fermé de la variable complexe z (sphère de Riemann) et passant par le point $z=\infty$. Désignons par $D(\gamma)$ l'un des domaines bornés par γ et par $w=f(z, \gamma)$, $f(\infty, \gamma)=\infty$, la fonction réalisant la représentation conforme du domaine $D(\gamma)$ sur le demi-plan inférieur $v<0$ du plan $w=u+iv$. Désignons par $V(s)$ les valeurs de $|f'(z, \gamma)|$ aux points de γ considérés en fonction de l'arc s de la courbe γ; la direction doit être choisie de telle manière que s croisse avec u.

Théorème 1. *Supposons que la tangente T à γ passant par l'un de ses points s_0 divise γ en deux parties situées de côtés opposés de T et que d'ailleurs au voisinage du point s_0 la ligne γ possède une courbure vérifiant la condition de Hölder. Dans ces conditions on a:*

$$V'(s_0)>0$$

ou bien

$$V'(s_0)<0$$

suivant que le rayon de la tangente T, issu de s_0 et dirigé du côté où s croît, est situé dans le domaine $D(\gamma)$ ou bien hors de ce domaine.

2. En conservant les notations du § 1, introduisons une notion géométrique. Supposons que la courbe γ possède en un point t_0 un cercle de courbure K de rayon r, le centre de K étant situé hors de $D(\gamma)$. Choisissons la direction positive sur γ de telle manière qu'au point t_0 elle coïncide avec le sens direct sur le cercle K. Désignons par z_0 le centre du cercle K et posons $\arg(t_0-z_0)=\alpha_0$, $0\leqslant\alpha_0<2\pi$. Désignons encore par $\alpha(t)$ la fonction continue du point t de la courbe γ qui coïncide (à un multiple de 2π près) avec l'argument de $(t-z_0)$ et qui est égale à α_0 au point t_0. Considérons l'ensemble $E^+(E^-)$ des points t de la courbe γ tels que 1) la direction du mouvement de t_0 vers t coïncide avec la direction positive (négative) sur γ, 1) pour chaque point t de notre ensemble la fonction $\alpha(t)$ est inférieure (supérieure) à α_0. Soit $m^+(m^-)$ la borne inférieure de valeurs de $|t-z_0|-r$ sur l'ensemble $E^+(E^-)$.

Soit, d'ailleurs, m^0 la borne inférieure des rayons des cercles contenus dans $D(\gamma)$ et tangents à γ au point t_0. Le nombre k égal au plus petit des trois nombres $\frac{m^+}{2r}$, $\frac{m^-}{2r}$, $\frac{m^0}{r}$ sera nommé degré d'accessibilité du point t_0. Il suit de la définition même du degré d'accessibilité que si le cercle K est situé hors du domaine $D(\gamma)$, le degré d'accessibilité est un nombre positif. Si la courbe γ est telle que chaque parallèle à une certaine direction fixe coupe γ en un point au plus, le degré d'accessibilité coïncide avec $\frac{m_0}{z}$.

Théorème 2. *Soit γ une courbe simple fermée; supposons qu'au voisinage de l'un de ses points s_0 la courbure est continue ainsi que ses dérivées des deux premiers ordres. Dans ces conditions, si le cercle de courbure pour le point s_0 est situé à l'intérieur de $D(\gamma)$, on a:*

$$\frac{d^2 \lg V(s_0)}{ds^2} > \frac{1}{4r^2}, \tag{1}$$

tandis que dans le cas où ce cercle est hors de $D(\gamma)$ on a:

$$\frac{d^2 \lg V(s_0)}{ds^2} < -\frac{C(k)}{r^2}, \tag{2}$$

où r est le rayon de courbure de γ au point s_0, et $C(k)$ est une constante positive qui ne dépend que du degré d'accessibilité k du point s_0.*

3. Les propriétés indiquées des fonctions univalentes peuvent être appliquées dans la théorie des sillages non seulement pour démontrer l'existence et l'unicité de la solution **, mais encore pour l'étude de cette solution au point de vue qualitatif. En complétant la Note précédente nous voulons indiquer quelques résultats de caractère qualitatif. Soit γ un arc à courbure bornée, symétrique par rapport à l'axe ox et tel que chaque parallèle à l'axe oy coupe γ en deux points au plus. Supposons que γ est situé dans un courant d'un fluide dont le potentiel complexe $w = f(z)$, $f'(-\infty) = 1$ est tel que

1) $f(\bar{z}) = f(z)$;

2) le long de γ et des courbes $y = \pm(x, \gamma)$ pour $y(x, \gamma) \geqslant 0$ on a: $\operatorname{Im} f(z) = 0$;

3) le long des courbes $y = \pm y(x, \gamma)$ pour $y(x, \gamma) > 0$ on a $|f'(z)| = 1$.

Dans ces conditions on a le théorème suivant.

Théorème 3. *Soient γ et $\bar{\gamma}$ deux arcs symétriques par rapport à l'axe des x, ayant des extrémités communes et tels que chaque parallèle à l'axe des y coupe γ et $\bar{\gamma}$ en deux points au plus. Si pour des abscisses égales les modules des ordonnées des points de $\bar{\gamma}$ sont toujours inférieurs ou égaux aux modules des ordonnées des points de γ, on aura:*

$$y(x, \bar{\gamma}) \geqslant y(x, \gamma),$$

le signe d'égalité n'ayant lieu que dans le cas où γ et $\bar{\gamma}$ coïncident.

En s'appuyant sur ce théorème et en tenant compte du lemme de la Note précédente il est bien facile de décrire du point de vue qualitatif les courants d'un fluide situés hors d'un corps convexe donné.

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* Dans ma Note précédemment citée (γ étant assujettie à certaines conditions restrictives supplémentaires) les inégalités (1) et (2) étaient par erreur écrites dans la forme $V''(s_0) > 0$ et $V''(s_0) < 0$, mais la première de ces inégalités suit de l'inégalité (1), tandis que la seconde n'est pas vraie dans le cas général, ce qui est facile à montrer par des exemples.

** Voir ma Note précédente.

Soit T un corps convexe symétrique par rapport à l'axe des x et borné par une ligne Γ continue et à tangente variant d'une manière continue. En désignant par x_0 et x_1, $x_0 < x_1$, les abscisses des points d'intersection de Γ avec l'axe des x, désignons par γ_t la partie de Γ appartenant à la bande $x_0 \leqslant x \leqslant t$ et par Γ_t la partie de Γ complémentaire à γ_t. Supposons que γ_t est situé dans un courant d'un fluide dont le potentiel complexe $w = f(z, t)$, $f'(+\infty, t) = 1$ est tel que 1) $f(z, t) = \overline{f(\bar{z}, t)}$, 2) le long de γ_t et des courbes $y = \pm y(x, \gamma_t) = \pm y(x, t)$, $x \geqslant t$, $y(x, t) \geqslant 0$, on a $\operatorname{Im} f(z, t) = 0$, 3) le long de la courbe $y = \pm y(x, t)$ pour $y(x, \gamma) > 0$ on a $|f'(z, t)| = 1$.

Théorème 4. *Il existe deux nombres t_1 et t_2, $x_0 < t_1 < t_2 < x_1$, dont le premier est inférieur aux abscisses des points de Γ à ordonnées extrêmales et tels que pour $x_0 < t < t_1$ les sillages $y = \pm y(x, t)$ coupent T, pour $t_1 \leqslant t \leqslant t_2$ les sillages $y = \pm y(x, t)$ ne coupent pas T et possèdent des branches infinies et pour $t_2 < t < x_1$ les sillages $y = \pm y(x, t)$ ne coupent pas T, et d'ailleurs pour $x \geqslant x(t) > x_1$ on a $y(x, t) \equiv 0$, les sillages se réunissent à une distance finie du corps.*

En conservant les mêmes notations, si l'on considère la vitesse à l'infini et la densité du fluide comme fixes, parmi tous les mouvements possibles d'un fluide ayant pour potentiel $f(z, t)$ le mouvement ayant pour potentiel $f(z, t_1)$ donnera la plus petite valeur à la résultante des forces de pression du fluide sur le corps. La valeur de cette résultante sera désignée par $P(T)$.

Considérons le rectangle $|x| \leqslant a$, $|y| \leqslant b$. Prenons sur le côté gauche de ce rectangle un segment d symétrique par rapport à l'axe des x et résolvons pour ce segment le problème de Kirchhoff. La dimension de d sera choisie de telle manière que les lignes $y = \pm y(x, d)$ passent par les points $a + ib$ et $a - ib$. Désignons par $T(a, b)$ le corps borné par le segment d, les lignes $y = \pm y(x, d)$ et le côté droit de notre rectangle.

Théorème 5. *Parmi tous les corps convexes que l'on peut inscrire dans le rectangle $|x| \leqslant a$, $|y| \leqslant b$,* le corps $T(a, b)$ donne la plus petite valeur à la fonctionnelle $P(T)$.

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Manuscrit reçu
le 28.V.1938.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

MATHÉMATIQUES

SUR LA THÉORIE DES SILLAGES

Par M. LAVRENTIEFF

(Présenté par I. M. Vinogradow, de l'Académie, le 27. V. 1938)

Dans deux Notes «Sur la théorie des sillages» (C. R. Acad. Sci. URSS, XVIII, № 4—5) et la précédente j'ai indiqué quelques nouvelles propriétés limites des fonctions réalisant la représentation conforme d'un domaine donné sur un demi-plan et j'ai indiqué quelques applications de ces résultats au problème de la théorie des sillages. Dans la présente Note je me propose d'indiquer quelques résultats analogues pour le cas des fonctions réalisant la représentation conforme d'un domaine sur une bande et de donner quelques applications hydrodynamiques de ces résultats.

1. Convenons d'admettre les notations suivantes. Soit $D(\gamma)$ un domaine simplement connexe borné par des courbes partiellement unies γ_1 et γ à extrémités communes aux points z_1 et z_2 *. Désignons par $w=f(z,\gamma,\gamma_1)$ une fonction réalisant la représentation conforme du domaine $D(\gamma)$ sur la bande $-1<v<0$ du plan $w=u+iv$ en supposant $f(z_1,\gamma,\gamma_1)=-\infty$, $f(z_2,\gamma,\gamma_1)=+\infty$. Désignons par $V(s)$ la valeur de $|f'(z,\gamma,\gamma_1)|$ aux points de γ considérée en fonction de la longueur de l'arc s de la courbe γ. Dans la suite nous supposerons γ et γ_1 tels que $V(s)$ existe et soit non nulle sur γ et sur γ_1.

On prend pour base des démonstrations des théorèmes qui suivent le lemme suivant:

Lemme. *Considérons deux domaines $D(\gamma)$ et $D(\bar{\gamma})$ bornés respectivement par les courbes γ_1, γ et γ_1, $\bar{\gamma}$. Supposons que $D(\bar{\gamma})$ contient $D(\gamma)$ et que les lignes γ et $\bar{\gamma}$ ont pour point commun le point z_0 (sans compter les points z_1 et z_2). Dans ces conditions on a:*

$$|f'(z_0,\bar{\gamma},\gamma_1)|\geqslant|f'(z_0,\gamma,\gamma_1)|$$

et d'ailleurs en chaque point z de l'arc γ_1

$$|f'(z,\bar{\gamma},\gamma_1)|\leqslant|f'(z,\gamma,\gamma_1)|,$$

où le signe d'égalité n'a lieu que dans le cas où γ et $\bar{\gamma}$ coïncident.

Énonçons maintenant deux résultats concernant l'allure de $V(s)$.

Théorème 1. *Soit γ_1 l'axe des x ou bien un segment de cet axe, et soit γ une courbe partiellement unie située dans le demi-plan supérieur*

* Les points z_1 et z_2 peuvent coïncider avec le point ∞.

n'ayant avec l'axe des x pour points communs que ses extrémités z_1 *et* z_2, $z_1 < z_2$, *et telle que* 1) *un certain point* s_0 *divise la courbe* γ *en deux parties situées entièrement de côtés opposés par rapport à la tangente T à* γ *passant par le point* s_0, 2) *au voisinage du point* s_0 *la courbe* γ *possède une courbure vérifiant la condition de Hölder. Dans ces conditions, si en suivant T à partir du point* s_0 *dans la direction où s croît nous arrivons dans le domaine* $D(\gamma)$ [dans le domaine complémentaire à $D(\gamma)$] *ou bien si nous restons toujours sur* γ *pour* $y > 0$, *nous aurons:*

$$V'(s_0) \geqslant 0 \qquad (V'(s_0) \leqslant 0), \tag{1}$$

les signes d'égalité n'ayant lieu que dans le cas, où γ *est une droite parallèle à l'axe des x.*

Si dans les conditions de l'inégalité (1) ou suppose encore que l'angle α formé par la direction positive de T * et la direction positive de l'axe ox est contenu entre 0 et $-\pi$, nous aurons respectivement

$$V'(s_0) \geqslant \frac{\sin^2\alpha}{|\alpha|}\frac{1}{y_0^2} \qquad \left(V'(s_0) \leqslant -\frac{\sin^2\alpha}{|\alpha|}\frac{1}{y_0^2}\right),$$

où y_0 est l'ordonnée du point s_0.

Théorème 2. *En conservant les notations précédentes, supposons qu'au voisinage de* s_0 *l'arc* γ *possède une courbure deux fois dérivable et que* γ_1 *est un arc jordanien, arbitraire. Si le cercle de courbure de* γ *au point* s_0 *appartient à* $D(\gamma)$ (est situé hors de $D(\gamma)$) *on a:*

$$\frac{d^2 \lg V(s_0)}{ds_0^2} > \frac{1}{4r^2} - \frac{dV(s_0)}{ds_0}\left(\frac{d^2 \lg V(s_0)}{ds_0^2} < -\frac{C(k)}{r^2} - \frac{dV(s_0)}{ds_0}\right),$$

où r est le rayon de courbure de γ *au point* s_0 *et* $C(k)$ *est une constante positive qui ne dépend que du degré d'accessibilité du point* s_0 **.

2. Les résultats que nous venons de citer permettent d'obtenir au moyen des méthodes directes du calcul des variations certains théorèmes d'existence et certains résultats qualitatifs dans la théorie générale des sillages. Indiquons un théorème d'existence:

Théorème 3. *Soit* γ_1 *une courbe située dans le demi-plan inférieur et définie par une fonction uniforme de* x: $y = F_1(x)$, $|x| < \infty$. *Soit d'ailleurs* Γ_1 *un arc donné:* $y = \varphi(x)$, *où* $\varphi(x)$ *est définie, uniforme et possède des dérivées des deux premiers ordres sur le segment* $(0, x_1)$, *et d'ailleurs* $\varphi(0) = 0$, $F_1(x) < \varphi(x) \leqslant 0$. *Dans ces conditions il existe une seule courbe* Γ_2: $y = \psi(x)$, $\psi(x_1) = \varphi(x_1)$, $\psi(x) \leqslant 0$ définie pour $x \geqslant x_1$ et telle qu'en chaque point de cette courbe pour $\psi(x) < 0$ on a:

$$V(s) = |f'(z, \gamma, \gamma_1)| = c = \text{const},$$

où γ est la courbe formée de la partie négative de l'axe des x, de l'arc Γ_1 et de la courbe Γ_2.

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Manuscrit reçu
le 28. V. 1938.

* La direction positive sur T doit être choisie de telle manière qu'elle coïncide au point s_0 avec la direction de la croissance de s.

** La définition du degré d'accessibilité est donnée dans ma Note précédente.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

MATHÉMATIQUES

SUR UN CRITÈRE DIFFÉRENTIEL DES TRANSFORMATIONS HOMÉOMORPHES DES DOMAINES À TROIS DIMENSIONS

Par M. LAVRENTIEFF

(Présenté par I. M. Vinogradow, de l'Académie, le 27. V. 1938)

Soit

$$u = u(x, y, z),\ v = v(x, y, z),\ w = w(x, y, z) \qquad (1)$$

une transformation continue de la sphère $S: x^2+y^2+z^2 < r^2,\ r \leqslant \infty$, en un domaine D de l'espace (u, v, w). Nous supposerons que les fonctions u, v, w sont dérivables dans la sphère S et que dans la même sphère on a pour le déterminant fonctionnel l'inégalité:

$$\frac{D(u, v, w)}{D(x, y, z)} > 0.$$

Dans ces conditions dans la transformation (1) il correspond à une sphère infiniment petite $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2 \leqslant \rho^2$ un ellipsoide infiniment petit dont a et c sont le grand et le petit demi-axe. Nous dirons que la transformation (1) est quasi-conforme dans la sphère S s'il existe une constante k, $k > 1$, telle que pour chaque point (x_0, y_0, z_0) de la sphère S on a:

$$\lim_{\rho \to 0} \frac{a}{c} \geqslant k.$$

On peut établir pour les transformations quasi-conformes une série de propriétés analogues à celle des transformations quasi-conformes des domaines plans. Indiquons deux propositions qui sont spécifiques pour le cas des domaines à trois dimensions.

Théorème 1. *Supposons que les fonctions* (1) *donnent la représentation quasi-conforme d'une sphère* S, $r < \infty$, *sur une sphère* $\sum: u^2+v^2+w^2 < r^2$, *ou bien sur une partie de cette sphère et que chaque point de la surface de la sphère* $\sum$ *est limite pour les points* (u, v, w) *correspondant aux points de* S. *Dans ces conditions les relations* (1) *réalisent une transformation homéomorphe entre les sphères fermées* S *et* $\sum$.

Théorème 2. *Supposons que la transformation* (1) *est une transformation quasi-conforme de l'espace* (x, y, z) *en* (u, v, w) *ou bien en une partie de l'espace* (u, v, w). *Dans ces conditions la transformation* (1) *est une transformation homéomorphe entre les espaces* (x, y, z) *et* (u, v, w).

Ce dernier théorème peut encore être énoncé de la manière suivante: si la transformation (1) est une transformation quasi-conforme de tout l'espace, le système (1) peut être résolu d'une manière unique par rapport à x, y, z pour toutes les valeurs de u, v, w.

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Manuscrit reçu
le 28. V. 1938.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4.

MATHEMATICS

ON SOME CRITERIA FOR THE REGULARITY OF SPACES OF THE TYPE (B)

By D. MILMAN

(Communicated by S. N. Bernstein, Member of the Academy, 31. V. 1938)

I. In the present note we accept the terminology and the denotations used by Banach in his well-known book (1).

We shall moreover introduce the following definitions:

The space E of the type (B) is called a regular space, if for every $x \in \overline{E}$ there exists an $X \in \overline{E}$ such that

$$f(x) = X(f) \text{ for all } f \in E.$$

A set of elements $\mathfrak{M}$ is called transfinitely closed, if for every transfinite bounded sequence $x_1, x_2, \ldots, x_\eta, \ldots (\eta < \vartheta)$ there exists at least one element x_0 (called the transfinite limit of the given sequence) such that

$$\varliminf_{\eta<\vartheta} f(x_\eta) \leqslant f(x_0) \leqslant \varlimsup_{\eta<\vartheta} f(x_\eta) \text{ for all } f \in \overline{E}.$$

A. Plessner has given the following criterion for the regularity of the space (2):

A) In order that a space * should be regular, it is necessary and sufficient that its unite sphere $\|x\| \leqslant 1$ should be transfinitely closed.

On the other hand, V. Šmulian has kindly informed me that the following proposition is true:

B) In order that a convex, closed set K should be transfinitely closed, it is necessary and sufficient that for every sequence of convex, closed and non-empty sets

$$K_1 \supseteq K_2 \supseteq \ldots \supseteq K_\eta \supseteq \ldots \quad (\eta < \vartheta)$$

of which each but the first is contained in the preceding one, there should exist an element x_0 belonging to all $K_\eta (\eta < \vartheta)$.

We give briefly the proof of this proposition.

Let $x_1, x_2, \ldots x_\eta, \ldots (\eta < \vartheta)$ be a transfinite bounded sequence of elements and let K_η denote the smallest convex, closed set containing $x_\eta, x_{\eta+1}, \ldots x_\xi, \ldots (\xi < \vartheta)$. It is easily seen that

$$\varliminf_{\eta<\vartheta} f(x_\eta) = \sup_{\eta<\vartheta} \inf_{x \in K_\eta} f(x); \quad \varlimsup_{\eta<\vartheta} f(x_\eta) = \inf_{\eta<\vartheta} \sup_{x \in K_\eta} f(x).$$

* We consider here and in what follows only spaces of the type (B).

On the other hand, an arbitrary element x_0 belongs to all K_η if and only if

$$\sup_{\eta<\vartheta} \inf_{x\in K_\eta} f(x) \leqslant f(x_0) \leqslant \inf_{\eta<\vartheta} \sup_{x\in K_\eta} f(x) \text{ for all } f\in \overline{E},$$

and the statement of Šmulian is proved.

Banach has proved that

C) A space $\overline{E}$ is regular, if it is separable and its unite sphere ($\|x\|\leqslant 1$) is weakly compact [see (1), pp. 189—190].

Gantmacher and Šmulian (2) have proved that

D) The unite sphere of every regular space is weakly compact.

II. In connection with the above propositions we prove the following.

Theorem 1. *In order that a space should be regular, it is necessary and sufficient that its unite sphere should be weakly compact and that every face of this sphere should be transfinitely closed.*

Hereby under a face of the sphere $\|x\|\leqslant r$ we understand a convex set of boundary points of this sphere which satisfy the equation $f(x)=\|f\|$ for a certain $f\in\overline{E}(f\neq 0)$.

Proof. The necessity of the conditions of the theorem follows directly from propositions A) and D).

In order to prove that if these conditions are satisfied the space will be regular, it is in virtue of the propositions A) and B) sufficient to show that in the given space *there exists for every transfinite sequence of convex, closed and non-empty sets* $\{K_\eta\}_{\eta<\vartheta}$, *containing each the following one, an element* x_0 *belonging to all* K_η.

Let a be an arbitrary point outside K_1. Applying Mazur's theorem that every convex, closed set is weakly closed, it is easily verified that the distance between any point and a convex, closed and weakly compact set is always attained. Let $x_\eta(\eta<\vartheta)$ be the point at which the distance from a to K_η is attained. Let

$$\|x_\eta - a\| = d_\eta(\eta<\vartheta);\ \sup_{\eta<\vartheta} d_\eta = d.$$

Obviously we have $d_{\eta_1}\leqslant d_{\eta_2}$ when $\eta_1\leqslant\eta_2$.

We consider the following two cases:

1) $d_\eta < d$ for every $\eta<\vartheta$.

Let $d_{\eta_1} < d_{\eta_2} < \ldots \longrightarrow d$. It is obvious that the sequence $\eta_1<\eta_2<\eta_3<\ldots$ defines a transfinite index ϑ.

Let x_0 be a weak condensation point of the sequence $\{x_{\eta_n}\}$: we can assume that

$$x_{\eta_n}\longrightarrow x_0 \quad \text{when} \quad n\longrightarrow\infty.$$

By Mazur's theorem we conclude that $x_0\in K_{\eta_p}(p=1,2,3\ldots)$.

Let η be an arbitrary index $<\vartheta$. There exists an integer N such that $d_\eta < d_{\eta_N}$; consequently $\eta<\eta_N$ and $x_0\in K_{\eta_N}\subset K_\eta$. Thus, x_0 is the required element.

2) $d_{\eta_0}=d$ for a certain $\eta_0<\vartheta$.

It is obvious that in this case $d=d_\eta$ for $\eta_0\leqslant\eta<\vartheta$. Let S be a sphere of radius $d=d_{\eta_0}$ with the centre a and $S_\eta = K_\eta\cdot S$ for all $\eta_0\leqslant\eta<\vartheta$. Evidently S_{η_0} is a part of a certain face of S. Consequently, by the condition of the theorem, S_{η_0} is transfinitely closed. But then there exists, according to B), an element x_0 belonging to all $S_\eta(\eta_0\leqslant\eta<\vartheta)$ and consequently to all $K_\eta(\eta<\vartheta)$ which proves the statement.

Corollary. *If the unite sphere of a given space is weakly compact and all the faces of this sphere are separable, then the space is regular.*

In fact every transfinite sequence of separable closed sets containing each the following one contains only an enumerable number of different sets.

Hence, by B) it is easily seen that every convex, closed, weakly compact and separable set is transfinitely closed. Consequently, under the conditions stated all the faces of the unite sphere are transfinitely closed.

Theorem 1 can be generalized in the following way.

Theorem 1a. *In order that a space should be regular, it is necessary and sufficient that its unite sphere should be weakly compact and that on the boundary of this sphere there should be at least one point y_0, in a certain neighbourhood of which every face of the sphere is transfinitely closed; in particular, if in the neighbourhood of a point on the boundary of the unite sphere every face is separable and the unite sphere itself is weakly compact, then the space is regular.*

In the present note we confine ourselves to the statement that the theorem can be proved in the same way provided that the point a is chosen in the following special way.

Let T_0 be a sphere of radius R_0 enclosing K_1, and let T_1 be a concentric sphere of radius $3R$.

Then we choose on the ray $y=-ty_0 (t>0)$ a point $y_1=-t_0y_0$ such that the boundary of the sphere with the centre at y_1 and radius t_0+3R_0 should contain in the neighbourhood of the point $3Ry_0$ enclosing the sphere T_1 only transfinitely closed faces. It is easily seen that such a choice is always possible.

III. Clarkson [3] has introduced the conception of uniform convexity of the space. A space is called uniformly convex, if for every $\varepsilon>0$ such $\delta>0$ can be found that from the inequality

$$\|x-y\|>\delta (\|x\|=\|y\|=1)$$

follows $\left\|\frac{x+y}{2}\right\|\leqslant 1-\varepsilon$.

We shall call a unite sphere uniformly convex in the r-neighbourhood of the point $x_0 (\|x_0\|=1)$, if for every $\varepsilon>0$ such $\delta>0$ can be found that from

$$\|x-y\|>\delta;\ \|x\|=\|y\|=1;\ \|x-x_0\|\leqslant r;\ \|y-y_0\|\leqslant r$$

there follows

$$\left\|\frac{x+y}{2}\right\|\leqslant 1-\varepsilon.$$

Applying the same method as in the proof of Theorem 1, we shall now prove the following.

Theorem 2. *Every uniformly convex space is regular.*

Proof. Let $K_1, K_2, \ldots K_\eta \ldots (\eta<\vartheta)$ be a transfinite sequence of convex, closed and non-empty sets containing each the following one. According to the remarke made above, the theorem will be proved if we shall show that whatever be the sequence, there will always exist a point belonging to all $K_\eta (\eta<\vartheta)$.

It is easily verified that *in a uniformly convex space the distance between a point and a convex closed set is always attained and moreover only in one point.*

Let a be an arbitrary point outside K_1; let the distance from a to $K5$ be attained in x_η. Let $\|a-x_\eta\|=d_\eta$; $\sup\limits_{\eta<\vartheta} d_\eta=d$.

Two cases are now possible:

1) $d_{\eta_0} = d\,(\eta_0 < \vartheta)$. Then in virtue of uniform convexity of the unite sphere it follows that x_{η_0} is the required element.

2) $d_{\eta_1} < d_{\eta_2} < \ldots \to d$ and there does not exist an $\eta < \vartheta$ for which $d_\eta = d$.

The sequence $x_{\eta_1}, x_{\eta_2}, \ldots$ has a strong limit x_0. In fact, let us assume that at the contrary

$$\|x_{\eta_{m_p}} - x_{\eta_{n_p}}\| \geqslant \delta > 0,$$

where $p = 1, 2, 3 \ldots$, $\eta_{n_1} < \eta_{m_1} < \eta_{n_2} < \eta_{m_2} \ldots$

Since $\lim\limits_{p\to\infty} \|x_{\eta_{m_p}}\| = \lim\limits_{p\to\infty} \|x_{\eta_{n_p}}\| = d$ and the space is uniformly convex, there exists such $\varepsilon > 0$ that

$$\left\| \frac{x_{\eta_{m_p}} + x_{\eta_{n_p}}}{2} \right\| \leqslant (1-\varepsilon)\, d \quad (p \geqslant N).$$

But $\frac{x_{\eta_{m_p}} + x_{\eta_{n_p}}}{2} \in K_{\eta_{n_p}}$; therefore

$$d_{\eta_{n_p}} \leqslant \left\| \frac{x_{\eta_{m_p}} + x_{\eta_{n_p}}}{2} \right\| \leqslant (1-\varepsilon)\, d$$

and we have come to a contradiction. Thus, there exists $\lim\limits_{n\to\infty} x_{\eta_n} = x_0$. Since the sequence $\eta_1, \eta_2, \ldots$ defines ϑ, we conclude, applying Mazur's theorem mentioned above, that $x_0 \in K_\eta$ for $\eta < \vartheta$. This proves the theorem.

From the last theorem in particular it follows that a part of Clarkson's ([3]) results on the differentiation of abstract functions form a corollary of the results obtained by I. Gelfand in his dissertation (1935) ([4]).

Theorem 2 admits the following generalization which can be proved in the same way as Theorem 1a.

Theorem 2a. *If on the boundary of the unite sphere of a space there exists at least one point y_0 in a certain neighbourhood $\|x - y_0\| \leqslant r$ of which the unite sphere is uniformly convex, then the space is regular.*

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Received
3. VI 1938.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

MATHÉMATIQUES

LES PROBLÈMES LIMITES POUR LES ÉQUATIONS DU TYPE ELLIPTICO-PARABOLIQUE

Par N. PISKOUNOV

(Présenté par I. M. Vinogradow, de l'Académie, le 27. V. 1938)

Il existe trois types d'équations différentielles linéaires du second ordre à N variables que l'on n'a presque pas étudiées jusqu'à présent. Ce sont les équations suivantes:

1. Les équations hyperboliques anormales — celles dont la forme quadratique contient après la réduction parmi N carrés n carrés à coefficients négatifs, $1 < n < N-1$.

2. Les équations elliptico-paraboliques — celles dont la forme quadratique contient après la réduction n_1 carrés à coefficients positifs, où $n_1+1 < N$.

3. Les équations hyperbolico-paraboliques — celles dont la forme quadratique contient après la réduction n_2 carrés, dont n_2' ont des coefficients positifs, n_2'' — des coefficients négatifs et $n_2 < N$.

Certaines propriétés des équations hyperboliques anormales ont été étudiées par Asgeirsson (1), et pour les équations du type hyporbolico-parabolique à trois variables quelques problèmes limites ont été considérés par l'auteur (2). Le fait que ces équations ont été peu étudiées est facile à expliquer: on ne connaît aucun problème physique dont la solution se réduit à l'étude de ces équations. D'ailleurs, par la même raison ces problèmes sont difficiles à résoudre: la physique ne donne aucune indication sur la direction des recherches concernant ces équations (quels sont les problèmes que l'on doit poser pour ces équations, quel est le domaine où les solutions peuvent être définies, les propriétés de ces solutions, etc.).

Il n'en est pas ainsi pour les équations elliptico-paraboliques. Ces équations ont été obtenues au cours de la résolution de problèmes physiques. Dans ses travaux «Zur Theorie der stetigen zufälligen Prozesse» (3) et «Zufällige Bewegungen» («Zur Theorie der Brownschen Bewegung») A. Kolmogoroff a montré que la loi de distribution pour les quantités qui ont fait l'objet de ces considérations est exprimée par une fonction qui vérifie une équation du type elliptico-parabolique (une équation du type parabolique dégénerée pour plusieurs variables). Certains problèmes hydro-dynamiques conduisent aussi aux équations de ce type. L'auteur a étudié le problème suivant: Détermination du débit d'un courant par une méthode chimique. Un cas particulier du problème considéré (pour le cas d'une équation du type elliptico-parabolique) a été étudié par l'auteur dans un article précédent (4).

Dans le présent article on considère l'équation

$$\frac{\partial^2 u}{\partial x^2} = \sum_{i=1}^{m} a_i \frac{\partial u}{\partial y_i} + c \frac{\partial u}{\partial x} + bu + f; \qquad \sum_{i=1}^{m} a_i^2 \geqslant \delta > 0, \tag{2}$$

où

$$\begin{aligned} a_i &= a_i(y_1 y_2 \ldots y_m, x), \\ c &= c\ (y_1 y_2 \ldots y_m, x), \\ b &= b\ (y_1 y_2 \ldots y_m, x), \\ f &= f\ (y_1 y_2 \ldots y_m, x). \end{aligned}$$

Les propriétés fondamentales de ces équations subsistent pour les équations de la forme

$$\sum_{k,\, l=1}^{n} A_{kl} \frac{\partial^2 u}{\partial x_k \partial x_l} + \sum_{k=1}^{n} B_k \frac{\partial u}{\partial x_k} = \sum_{i=1}^{m} a_i \frac{\partial u}{\partial y_i} + bu + f, \tag{1}$$

ce qui est très facile à démontrer.

Considérons dans l'espace $\{y_1 y_2 \ldots y_m, x\}$ un certain domaine V borné par une surface S. Supposons pour simplifier les raisonnements que S est homéomorphe à une sphère. Supposons d'ailleurs que chaque parallèle à l'axe ox passant par un point intérieur à V coupe S en deux points. Considérons ensuite l'ensemble des points d'intersection de la famille des droites parallèles à l'axe ox passant par tous les points intérieurs à V avec la surface S. Soit S_1' cet ensemble. Alore S_1' diffère de S par l'ensemble des points communs à S et au cylindre tangent à V, et dont les génératrices sont parallèles à l'axe ox. Désignons cet ensemble par S'.

Considérons la famille des courbes intégrales d'un système d'équations différentielles ordinaires

$$\frac{dy_1}{a_1} = \frac{dy_2}{a_2} = \ldots \frac{dy_m}{a_m}.$$

Ces lignes seront nommées lignes (β). Supposons qu'une certaine surface cylindrique à $(m-1)$ dimensions à génératrices parallèles à ox et homéomorphe à un cylindre à $(m-1)$ dimensions coupe l'ensemble en deux parties S_1' et S_2' de la manière suivante: les lignes (β) entrent dans V par S_1' sans être tangentes à S_1'; elles n'entrent pas par S_2'; la direction dans laquelle les lignes entrent sera définie par la condition d'avoir pour cosinus directeurs les valeurs

$$\frac{a_i}{+\sqrt{\sum_{i=1}^{m} a_i^2}}.$$

Nous allons considérer deux cas:

I. L'ensemble S_1' est un continu à $(m-1)$ dimensions ne contenant pas de segments parallèles à l'axe ox.

II. L'ensemble S_1' est une surface à m dimensions, cylindrique par rapport à x. Nous allons supposer que S_1' et S_2' sont homéomorphes à une sphère. Alors le problème se pose de la manière suivante.

En faisant certaines hypothèses concernant la régularité de a_i, b, c, f et S et la disposition réciproque des lignes (β) et de S, démontrer l'existence d'une fonction unique $u(y_1, y_2 \ldots y_m, x)$ vérifiant à l'intérieur du domaine V l'équation (2) et prenant sur les ensembles S_1' et S_1'' des valeurs données ($S_1' + S_1'' = S_1$).

Considérons le cas II.

Supposons que dans l'espace à $m+1$ dimensions $(y_1 y_2 \ldots y_m, x)$ on a un domaine V borné par les surfaces

$$\Phi_1(y_1, y_2, \ldots y_m) = 0, \quad \text{(a)}$$
$$\Phi_2(y_1, y_2, \ldots y_m) = 0, \quad \text{(b)}$$
$$x = \varphi_1(y_1, y_2, \ldots y_m), \quad \text{(c)}$$
$$x = \varphi_2(y_1, y_2, \ldots y_m). \quad \text{(d)}$$

Supposons que Φ_1, φ_1, φ_2 sont des fonctions continues admettant des dérivées continues des trois premiers ordres. On mène toutes ces surfaces de manière à vérifier les conditions suivantes:

1. Elles bornent une certaine partie de l'espace.

2. Les lignes (β) entrent dans le domaine V par la surface (a) sans être tangentes et n'entrent pas par la surface (b).

3. On a $\varphi_1(y_1, y_2 \ldots y_m) - \varphi_2(y_1, y_2, \ldots y_m) \geqslant \delta > 0$ partout dans le domaine borné par les surfaces (a) et (b), mais sur la surface (b) elle-même cette différence peut s'annuler.

Supposons que les coefficients a_i, b, c, f sont des fonctions continues possédant des dérivées des trois premiers ordres par rapport à leurs arguments. Alors on a le théorème suivant:

Théorème. *Si les fonctions* $f_1(y_1, y_2 \ldots y_m, x)$, $f_2(y_1, y_2 \ldots y_m, x)$, $f_3(y_1, y_2 \ldots y_m, x)$ *sont définies sur les surfaces* (a), (d) *et* (c) *et si d'ailleurs* f_1 *est une fonction continue par rapport à* x *et possède par rapport aux autres arguments des dérivées des trois premiers ordres, tandis que* f_2 *et* f_3 *possèdent des dérivées des trois premiers ordres, alors il existe une fonction* $u(y_1, y_2 \ldots y_m, x)$ *vérifiant l'équation* (2) *et coïncidant sur les surfaces* (a), (d) *et* (e) *avec les valeurs des fonctions* f_1, f_2, f_3. *La fonction vérifiant ces conditions est unique.*

Considérons le cas I.

Supposons que S_1' est un continu à $(m-1)$ dimensions sans segments verticaux. Désignons par $x=0$ sa projection sur le plan L_1. Après avoir fait la transformation $\bar{x} = x - \varphi(y_1, \ldots y_m)$, où $x = \varphi(y_1, \ldots y_m)$ est l'équation de la surface sur laquelle est située la courbe S_1', on voit que cette surface sera située dans le plan $x=0$ et coïncide avec la ligne L_1. Cette transformation effectuée, l'ensemble S_2' aura pour intersection avec le plan $x=0$ la courbe L_2 (S_2' peut être un ensemble à m dimensions). Transformons maintenant le plan $x=0$ de telle manière que L_1 se trouve dans le plan à $(m-1)$ dimensions $x=0$, $y_1=0$. Toutes ces transformations n'ont aucune influence sur la forme de l'équation (2). Le domaine V et les ensembles S_1'', S_1', S_2' définis au commencement du paragraphe se transforment en d'autres, mais les propriétés qui les caractérisaient subsistent. C'est pourquoi nous ne changerons pas leurs notations, excepté L_1, transformation de S_1'. La frontière de L_1 commune avec L_2 sera désignée par N.

Supposons que pour l'équation obtenue on a l'inégalité $a_1 \geqslant \delta > 0$ partout dans le domaine V sauf peut-être en un voisinage de l'ensemble N aussi petit que l'on veut, où a_1 peut s'annuler tandis que $a_i \neq 0$ $(i = 2, \ldots m)$. En vertu des hypothèses faites les lignes (β) entreront dans le domaine par L_1 et n'entreront pas par S_2'.

Alors, dans certaines hypothèses sur la régularité de S_1, on a le théorème suivant:

Théorème. *Si* S_1 *est une fonction définie sur* $u(S)$ *et possédant des dérivées des trois premiers ordres par rapport à ses arguments, il existe une fonction* $u(y_1, y_2, \ldots y_m, x)$ *vérifiant l'équation* (2) *dans le domaine*

V et coïncidant avec $u(S)$ sur la surface S_1. La fonction vérifiant ces conditions est unique.

Considérons encore le domaine V borné par les surfaces (a), (d), (e), (b). Supposons que $U_0(y_2, \ldots y_m, x)$, $f_1(y_1, y_2, \ldots y_m)$, $f_2(y_1, \ldots y_m)$ sont des fonctions assez régulières définies sur les surfaces (a), (d) et (c). Alors on a le théorème suivant:

Théorème. *Il existe dans le domaine V une et une seule fonction $U(y_1, y_2, \ldots y_m, x)$ continue ainsi que ses dérivées premières par rapport à y_i et ses deux premières dérivées par rapport à x, vérifiant l'équation* (2) *et les conditions limites*:

$$U(0, y_2, \ldots y_m, x) = U_0(y_2, \ldots y_m, x); \quad \frac{\partial U}{\partial x}_{x=\varphi_1} = f_1(y_1, \ldots y_m);$$

$$\frac{\partial U}{\partial x}_{x=\varphi_2} = f_2(y_1, \ldots y_m).$$

Cas d'un domaine infini. Considérons le domaine infini $0 \leqslant y_1 \leqslant \leqslant Y - \infty \leqslant x \leqslant + \infty$ et l'équation (2).

Supposons qu'on a dans ce domaine $a_1 \geqslant k > 0 \,|\, a_i \,| < \infty \,(i = 2 \ldots m)$. Alors, sans restreindre la généralité on peut supposer que $a_1 = 1$.

Théorème. *Soient $U_0(y_2, \ldots y_m, x)$, $P_1(y_1, y_2, \ldots y_m)$, $P_2(y_1, \ldots y_m)$ des fonctions données. Si dans l'équation* (2) *les fonctions a_i, b, f possèdent des dérivées jusqu'au second ordre par rapport à leurs arguments, P_1 et P_2 ont des dérivées jusqu'au troisième ordre, U_0 des dérivées des deux premiers ordres par rapport à y_i et des dérivées jusqu'au quatrième ordre par rapport à x; si, d'ailleurs, U_0 vérifie la condition $\lim_{x \to \pm \infty} \frac{\partial^2 U}{\partial x^2} = 0$, alors il existe une et une seule fonction $U(y_1, \ldots y_m, x)$ vérifiant l'équation* (2) *et les conditions*

$$U(0, y_2, \ldots y_m, x) = U_0(y_2, \ldots y_m, x); \lim_{x \to + \infty} U(y_1, \ldots y_m, x) = P_1(y_1, \ldots y_m);$$

$$\lim_{x \to + \infty} U(y_1, \ldots y_m, x) = P_2(y_1, \ldots y_m).$$

On suppose d'ailleurs que

$$\lim_{x \to + \infty} U_0(y_2, \ldots y_m, x) = P_1(0, y_2, \ldots y_m);$$

$$\lim_{x \to - \infty} U_0(y_2, \ldots y_m, x) = P_2(0, y_2, \ldots y_m);$$

$$\lim_{x \to \pm \infty} \frac{\partial^2 (U_0 - P)}{\partial y_i \partial y_j} = 0;$$

et le membre indépendant de U vérifie la condition

$$\lim_{x \to \pm \infty} f = 0.$$

On démontre que si cette dernière condition n'est pas vérifiée, la solution peut ne pas exister.

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Manuscrit reçu
le 1. VI. 1938.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

THEORY OF ELASTICITY

ON THE TORSION OF NEARLY PRISMATIC RODS

By D. PANOV

(Communicated by I. M. Vinogradow, Member of the Academy, 3. V. 1938)

1. In this note we give a solution of the problem of the torsion of a non-prismatic body which allows to carry out effectively the necessary calculations. The possibility of such a solution can be of course obtained only at the cost of some limitations: we suppose that the body under consideration is of a nearly prismatic form. From the point of view of applications this limitation is quite natural; it is for instance satisfied for the airscrew blade. Belov we shall formulate the assumptions, which we have to make, in a more presise form.

2. We consider the body S limited in a rectangular system of cartesian co-ordinates by the surface:

$$f[x(1-kz),\quad y(1-kz)]=0, \tag{1}$$

and by two planes perpendicular to the axis oz. In (1) k means a parameter whose squares and higher powers we shall neglect. We also assume that for the whole of the body under consideration

$$1-kz>0.$$

Putting

$$\left.\begin{aligned}\xi&=x(1-kz)\\ \eta&=y(1-kz)\\ \zeta&=z,\end{aligned}\right\} \tag{2}$$

we can reduce the solution of the problem for the body S to the solution of the problem for the prismatic body whose lateral surface is defined by the equation

$$f(\xi,\eta)=0. \tag{3}$$

We shall solve the problem of torsion of the body S and we must begin by defining what we mean by the term «torsion» when applied to the deformation of this body.

As is well known, the torsion of a prismatic body is called such a deformation of it which corresponds to the state of stress characterized by the equations:

$$X_x=Y_y=Z_z=X_y=0,$$

and by the absence of both volume forces and stresses on the lateral surface. The tangent stresses X_z and Y_z produce hereby a moment about the axis of the rod and are supposed to be distributed upon the bases in the same manner as in every cross-section.

Having in view to consider the torsion of a non-prismatic body, we shall agree to understand under its torsion such a deformation which reduces for $k=0$ to the torsion of the corresponding prismatic body.

The stresses on the surface will be suppose to be absent and the stresses in the bases will be obtained in the course of the solution. It can be shown that for the problem under consideration there exists only one such solution.

3. Transforming the usual equations of the theory of elasticity expressed in terms of displacements by means of the equations (2), we obtain, neglecting squares and higher powers of k:

$$\left.\begin{aligned}
(\lambda+\mu)\frac{\partial\theta}{\partial\xi}+\mu\Delta U &= k\Big[(\lambda+\mu)\Big(2\zeta\frac{\partial^2 U}{\partial\xi^2}+2\zeta\frac{\partial^2 V}{\partial\xi\partial\eta}+\zeta\frac{\partial^2 W}{\partial\xi\partial\zeta}+\\
&+\eta\frac{\partial^2 W}{\partial\xi\partial\eta}+\xi\frac{\partial^2 W}{\partial\xi^2}+\frac{\partial W}{\partial\xi}\Big)+2\mu\Big(\zeta\frac{\partial^2 U}{\partial\xi^2}+\\
&+\zeta\frac{\partial^2 U}{\partial\eta^2}+\xi\frac{\partial^2 U}{\partial\xi\partial\zeta}+\eta\frac{\partial^2 U}{\partial\eta\partial\zeta}\Big)\Big];\\
(\lambda+\mu)\frac{\partial\theta}{\partial\eta}+\mu\Delta V &= k\Big[(\lambda+\mu)\Big(2\zeta\frac{\partial^2 U}{\partial\xi\partial\eta}+2\zeta\frac{\partial^2 V}{\partial\eta^2}+\zeta\frac{\partial^2 W}{\partial\eta\partial\zeta}+\\
&+\xi\frac{\partial^2 W}{\partial\xi\partial\eta}+\eta\frac{\partial^2 W}{\partial\eta^2}+\frac{\partial W}{\partial\eta}\Big)+2\mu\Big(\zeta\frac{\partial^2 V}{\partial\xi^2}+\\
&+\zeta\frac{\partial^2 V}{\partial\eta^2}+\xi\frac{\partial^2 V}{\partial\xi\partial\zeta}+\eta\frac{\partial^2 V}{\partial\eta\partial\zeta}\Big)\Big];\\
(\lambda+\mu)\frac{\partial\theta}{\partial\zeta}+\mu\Delta W &= k\Big[(\lambda+\mu)\Big(\zeta\frac{\partial^2 U}{\partial\xi\partial\zeta}+\eta\frac{\partial^2 U}{\partial\xi\partial\eta}+\xi\frac{\partial^2 U}{\partial\xi^2}+\frac{\partial U}{\partial\xi}+\\
&+\zeta\frac{\partial^2 V}{\partial\eta\partial\zeta}+\xi\frac{\partial^2 V}{\partial\xi\partial\eta}+\eta\frac{\partial^2 V}{\partial\eta^2}+\frac{\partial V}{\partial\eta}+2\xi\frac{\partial^2 W}{\partial\xi\partial\zeta}+\\
&+2\eta\frac{\partial^2 W}{\partial\eta\partial\zeta}\Big)+2\mu\Big(\zeta\frac{\partial^2 W}{\partial\xi^2}+\zeta\frac{\partial^2 W}{\partial\eta^2}+\xi\frac{\partial^2 W}{\partial\xi\partial\zeta}+\\
&+\eta\frac{\partial^2 W}{\partial\eta\partial\zeta}\Big)\Big].
\end{aligned}\right\}\qquad(4)$$

Here U, V and W mean the displacements u, v and w in the co-ordinates ξ, η and ζ. For the boundary conditions on the surface (3) we obtain:

$$\left.\begin{aligned}
\lambda\theta\cos\alpha+\mu\Big(\frac{\partial U}{\partial\nu}+\frac{\partial U}{\partial\xi}\cos\alpha+\frac{\partial V}{\partial\xi}\cos\beta\Big) &= k\Big[\lambda\cos\alpha\Big(\zeta\frac{\partial U}{\partial\xi}+\\
&+\zeta\frac{\partial V}{\partial\eta}+\xi\frac{\partial W}{\partial\xi}+\eta\frac{\partial W}{\partial\eta}\Big)+\mu\Big(2\zeta\frac{\partial U}{\partial\xi}\cos\alpha+\zeta\frac{\partial U}{\partial\eta}\cos\beta+\\
&+\xi\frac{\partial U}{\partial\zeta}\cos\alpha+\eta\frac{\partial U}{\partial\zeta}\cos\beta+\zeta\frac{\partial V}{\partial\xi}\cos\beta+\xi\frac{\partial W}{\partial\xi}\cos\alpha+\\
&+\eta\frac{\partial W}{\partial\xi}\cos\beta\Big)\Big];\\
\lambda\theta\cos\beta+\mu\Big(\frac{\partial V}{\partial\nu}+\frac{\partial U}{\partial\eta}\cos\alpha+\frac{\partial V}{\partial\eta}\cos\beta\Big) &= k\Big[\lambda\cos\beta\Big(\zeta\frac{\partial U}{\partial\xi}+\\
&+\zeta\frac{\partial V}{\partial\eta}+\xi\frac{\partial W}{\partial\xi}+\eta\frac{\partial W}{\partial\eta}\Big)+\mu\Big(\zeta\frac{\partial V}{\partial\xi}\cos\alpha+2\zeta\frac{\partial V}{\partial\eta}\cos\beta+\\
&+\xi\frac{\partial V}{\partial\zeta}\cos\alpha+\eta\frac{\partial V}{\partial\zeta}\cos\beta+\zeta\frac{\partial U}{\partial\eta}\cos\alpha+\xi\frac{\partial W}{\partial\eta}\cos\alpha+\\
&+\eta\frac{\partial W}{\partial\eta}\cos\beta\Big)\Big];\\
\mu\Big(\frac{\partial W}{\partial\nu}+\frac{\partial U}{\partial\zeta}\cos\alpha+\frac{\partial V}{\partial\xi}\cos\beta\Big) &= k\Big[\lambda\theta(\xi\cos\alpha+\\
&+\eta\cos\beta)+\mu\Big(\zeta\frac{\partial W}{\partial\xi}\cos\alpha+\zeta\frac{\partial W}{\partial\eta}\cos\beta+2\xi\frac{\partial W}{\partial\zeta}\cos\alpha+\\
&+2\eta\frac{\partial W}{\partial\zeta}\cos\beta+\xi\frac{\partial U}{\partial\xi}\cos\alpha+\eta\frac{\partial U}{\partial\eta}\cos\alpha+\xi\frac{\partial V}{\partial\xi}\cos\beta+\\
&+\eta\frac{\partial V}{\partial\eta}\cos\beta\Big)\Big].
\end{aligned}\right\}\qquad(5)$$

It can be shown that the system of equations (4) and the boundary conditions (5) are satisfied up to the higher powers of k by the following functions U, V and W:

$$U = -\tau\eta\zeta + k\tau\left\{\Phi\xi - \frac{1}{2}(\eta\xi^2 + \eta^3) + \eta\zeta^2 - \frac{\mu}{\lambda+\mu}\left[\int^{\xi}\Phi d\xi + h_1(\eta)\right]\right\};$$

$$V = \tau\xi\zeta + k\tau\left\{\Phi\eta + \frac{1}{2}(\xi\eta^2 + \xi^3) - \xi\zeta^2 - \frac{\mu}{\lambda+\mu}\left[\int^{\eta}\Phi d\eta + h_2(\xi)\right]\right\};$$

$$W = \tau\Phi - 2k\tau\zeta\Phi.$$

In these formulae $\Phi = \Phi(\xi, \eta)$ denotes the torque function for the contour (3) and $h_1(\eta)$ and $h_2(\xi)$ are chosen in such a way that

$$\frac{\partial}{\partial\eta}\left[\int^{\xi}\Phi d\xi + h_1(\eta)\right] + \frac{\partial}{\partial\xi}\left[\int^{\eta}\Phi d\eta + h_2(\xi)\right] = 0.$$

In view of the properties of the function $\Phi(\xi, \eta)$ such choice is possible.

Passing back to the co-ordinates x, y, z, we obtain the following expressions for the displacements in the body S subjected to torsion, which are exact up to the higher powers of k:

$$u = -\tau yz(1-2kz) - \frac{1}{2}k\tau y(x^2+y^2) + k\tau\left\{x\Phi(x,y) - (1-2\sigma)\left[\int^{x}\Phi(x,y)\,dx + h_1(y)\right]\right\};$$

$$v = \tau xz(1-2kz) + \frac{1}{2}k\tau x(x^2+y^2) + k\tau\left\{y\Phi(x,y) - (1-2\sigma)\left[\int^{y}\Phi(x,y)\,dy + h_2(x)\right]\right\};$$

$$w = \tau\Phi(x,y)(1-2kz) - k\tau z\left(x\frac{\partial\Phi}{\partial x} + y\frac{\partial\Phi}{\partial y}\right).$$

Calculating the stresses corresponding to these displacements and computing the moment of the tangent stresses X_z and Y_z about the axis oz we can show that this moment does not depend on z, so that the obtained deformation can be indeed considered as torsion.

4. Putting

$$f(\xi, \eta) = \xi^2 + \eta^2 - a^2,$$

we obtain up to the squares and higher powers of k the known [1] exact solution of the problem of torsion of a cone.

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Received
25. V. 1928.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

THEORY OF ELASTICITY

SECONDARY EFFECTS ACCOMPANYING THE TORSION OF A CIRCULAR CYLINDER

By P. RIZ

(Communicated by I. M. Vinogradow, Member of the Academy, 23. V. 1938)

The present note deals with those phenomena, arising during the torsion of a circular cylinder, which cannot be investigated by the linear theory of elasticity neglecting the squares of the displacement derivatives. This problem has been already considered by Seth (1) who assumed the validity of the following form of Hook's law:

$$\left.\begin{aligned} \sigma_{xx} &= \lambda \sum_{\alpha=1}^{3} \varepsilon_{\alpha\alpha} + 2G\varepsilon_{xx}, \\ \sigma_{xy} &= G\varepsilon_{xy}, \end{aligned}\right\} \qquad (1)$$

where ε_{xx}, ε_{yy} are the deformation components with respect to the final state.

In a preceding paper by the author and N. Zvolinsky (2) it was shown that if we retain the second order terms in the displacement derivatives, Hook's law in the form (1) ceases to be valid, and a method for dealing with such problems * was suggested.

The present note accepts the formulation of the problem given by Seth, but uses the method described in our paper. Concerning the displacements we assume that:

$$\left.\begin{aligned} u &= x[1-\beta(r)\cos\tau z] - y\beta(r)\sin\tau z, \\ v &= y[1-\beta(r)\cos\tau z] + x\beta(r)\sin\tau z, \\ w &= \tau^2 \alpha z. \end{aligned}\right\} \qquad (2)$$

This is equivalent to assuming that a) the cross-sections remain plane, b) the angle of twist is proportional to z, and c) the lateral surface transforms again into the surface of a circular cylinder, but of a changed radius (x, y, z are the co-ordinates in the final state).

* This method is to a great extent based upon the results obtained by Murnaghan (3).

We solve the problem retaining the terms of the order not higher than τ^2, and put

$$\beta = 1 + \tau^2 \varphi(r).$$

For the deformation components we obtain the following expressions

$$\left.\begin{aligned} &\varepsilon_{xx} = -\tau^2\left(\varphi + \frac{x^2}{r}\varphi'\right);\quad \varepsilon_{yy} = -\tau^2\left(\varphi + \frac{y^2}{r}\varphi'\right);\quad \varepsilon_{zz} = \tau^2\left(\alpha - \frac{r^2}{2}\right);\\ &\varepsilon_{xy} = -xy\,\frac{2\tau^2\varphi'}{r};\quad \varepsilon_{xz} = -\tau y;\quad \varepsilon_{yz} = \tau x. \end{aligned}\right\} \tag{3}$$

Hence, using the method referred to above, we obtain the stresses:

$$\left.\begin{aligned} \sigma_{xx} &= \lambda\tau^2\left[\alpha - \left(r\varphi' + 2\varphi + \frac{r^2}{2}\right)\right] - 2G\tau^2\left(\varphi + \frac{x^2\varphi'}{r}\right) - \\ &\quad - \frac{m}{4}\tau^2 r^2 - \frac{n}{4}\tau^2 x^2 - G\tau^2 y^2;\\ \sigma_{yy} &= \lambda\tau^2\left[\alpha - \left(r\varphi' + 2\varphi + \frac{r^2}{2}\right)\right] - 2G\tau^2\left(\varphi + \frac{y^2\varphi'}{r}\right) - \\ &\quad - \frac{m}{4}\tau^2 r^2 - \frac{n}{4}\tau^2 y^2 - G\tau^2 x^2;\\ \sigma_{zz} &= \lambda\tau^2\left[\alpha - \left(r\varphi' + 2\varphi + \frac{r^2}{2}\right)\right] + 2G\tau^2\left(\alpha - \frac{r^2}{2}\right) - \\ &\quad - \frac{m}{4}\tau^2 r^2 - G\tau^2 r^2;\\ \sigma_{xy} &= -\tau^2 xy\left[\left(\frac{2\varphi'}{r} - 1\right)G + \frac{n}{2}\right];\\ \sigma_{xz} &= -G\tau y;\ \sigma_{yz} = G\tau x. \end{aligned}\right\} \tag{4}$$

Here $m = G\frac{9 - 12\nu}{1 - 2\nu}$, $n = 9G$ and ν is the Poisson ratio.

The first and the second body-stress equations are reduced to

$$\varphi'' + \frac{3\varphi'}{r} = A, \tag{5}$$

where

$$A = -\frac{7 - 16\nu}{4(1 - \nu)}.$$

Let us consider a particular solution, specified with the following conditions $\varphi(0) = 0$:

$$\varphi = \frac{A}{8} r^2. \tag{6}$$

The third body-stress equation is satisfied automatically.

The boundary conditions for the lateral surface, free from stresses, allow to determine the constant

$$\alpha = -\frac{3 + 3\nu - 12\nu^2}{8(1 - \nu)(1 - 2\nu)} R^2, \tag{7}$$

where R is the radius of the deformed cylinder.

For the value τ and the torque moment M we naturally obtain the usual relation:

$$M = G\tau \int\int (x^2 + y^2)\, dx\, dy,$$

but here the integral expresses the moment of inertia of the deformed cross-section.

In order to obtain the given displacements the normal stresses

$$\sigma_{zz} = \tau^2 (Br^2 - CR^2) \qquad (8)$$

should be distributed over the end sections of the cylinder; hereby

$$B = \frac{(1 + 10\nu - 20\nu^2)}{4(1-\nu)(1-2\nu)} G, \quad C = \frac{3G(1 + \nu - 4\nu^2)}{4(1-2\nu)^2}.$$

From the above it follows that 1) the twisted cylinder is compressed along its axis (this compression is characterized by the constant α), 2) at twisting the radius of the cylinder is increased in proportion

$$\frac{R_0}{R} = 1 - \tau^2 \frac{7 - 16\nu}{32(1-\nu)} R^2.$$

In conclusion it should be pointed out, that in spite of the complicated form of Hook's law as compared with the form accepted by Seth, the body-stress equation and all the basic formulae are obtained in a simpler form. We do not give here a detailed comparison of our results with those of Seth, but it should be noted that the modification of the radius obtained by Seth is proportional to the fourth degree of $R\tau$.

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Received
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

HYDRODYNAMICS

ON THE EXPERIMENTAL STUDY OF TURBULENCE OF REAL WATER OBJECTS

By O. BLUMBERG

(Communicated by O. Schmidt, Member of the Academy, 16. V. 1938)

The turbulence of natural water currents and reservoirs has been scarcely studied at all. The majority of papers on this subject arrives at its conclusions by indirect methods, in particular on ground of observations of temperature and salinity stratification. In comparatively few cases current-meters with chronographical record are used for the determination of the turbulency, whereby only the longitudinal component of the velocity $v \cos \beta \cos \gamma$ is usually obtained (v is the velocity scalar and β and γ are the angles between the velocity vector and the vertical and the horizontal plane passing through the dynamical axis). The variations of the angle β are recorded much more seldom, e. g. by means of the Eckmann-Mertz's current-meter, which gives the value of the transversal component $v \sin \beta \cos \gamma$. The existing current-meters, with the exception of one or two special designs, cannot be used ([1]) for the determination of the vertical component $v \sin \gamma$, so that this component, which is very important for the practical problems connected with sedimentation, bottom ice, etc., could not be to the present properly studied.

The method of velocity determination by means of coloured clouds developed by the author in 1933 ([2]) allows also to determine the turbulence of natural river flow. When the colour is introduced into the water instantaneously and without pressure, which can be achieved by a special apparatus, there arises in the water a coloured cloud having the form of a rotation ellipsoid which increases with time and moves with the current. The observations of the maximal dimensions of the cloud in three directions, namely $2R_{\text{long}}$, $2R_{\text{trans}}$ and $2R_{\text{vert}}$, allow not only to determine the angles β and γ by means of the formulae $R_{\text{vert}} = 2vT \sin \gamma_{\max}$ and $R_{\text{trans}} = 2vT \sin \beta_{\max}$, but also to separate the effect of the variation of the angles from the variations of the velocity scalar itself, and so to solve one of the fundamental problems of river hydraulics.

If we consider these latter additions to the mean value of the scalar $\pm \varepsilon$ as possible, we obtain (Fig. 1) the following approximative formulae (assuming the equality of the angles β and γ):

$$\Delta x = (v_{\max} - v_{\min} \cos^2 \beta)\, T,$$

$$\frac{\Delta y}{2v_{\max}} = T \sin \beta,$$

$$\frac{\Delta x}{\Delta y} = \frac{1}{2 \sin \beta} - \frac{v_{\min}}{v_{\max}} \frac{\cos^2 \beta}{2 \sin \beta},$$

whence

$$\frac{v_{\text{mean}}-\varepsilon}{v_{\text{mean}}+\varepsilon}=\frac{1-\frac{\Delta x}{\Delta y}2\sin\beta}{\cos^2\beta}.$$

The same characteristics may be obtained from the observations of the variations in time of the colour concentration at a point chosen on the stream axis. The same considerations as in the first case lead us (Fig. 2) to the relations:

$$\Delta T_{\text{rise}}=T_{\text{max concentr.}}-T_{\text{pioneer}}=\frac{L'}{v_{\text{mean}}}-\frac{L}{v_{\text{mean}}+\varepsilon}=\frac{\varepsilon L}{v_{\text{mean}}(v_{\text{mean}}+\varepsilon)};$$

$$\frac{\Delta T_{\text{rise}}}{T_{\text{max concentr.}}}=\frac{\varepsilon}{v_{\text{max}}};\qquad \cos^2\beta=\frac{1-\frac{\Delta T_{\text{fall}}}{T_{\text{final}}}}{1-\frac{\Delta T_{\text{rise}}}{T_{\text{pioneer}}}}.$$

The observation of the decrease of the maximum of concentration of coloured flood at two consecutive points represented by the higher hyper-

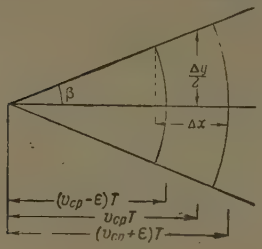

Fig. 1.

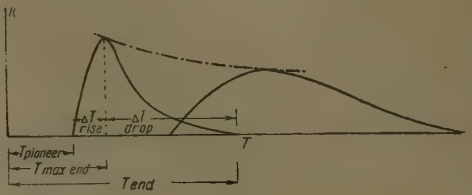

Fig. 2.

bolic curve $K_{\max}=C(p+sL)^{-m}$, enables us by means of the relation $R=bL^n$ to determine more exactly the turbulence, i. e. the character and the distribution of the angles β and γ within their maximal limits. We can pass from these angles to the more widely used characteristic of turbulence, namely the coefficient of turbulent exchange, by means of the author's approximative formula

$$A=\frac{K_{\text{mean}}}{K_{\max}}\frac{R^2}{3T}.$$

We can obtain a not less comprehensive picture of all variations of the velocity vector (moreover, in two planes) using the following method which scarcely requires any apparatus.

Let us take some sand of a definite (and quite considerable) hydraulic size according to the given longitudinal velocity of the current. It can be prepared by using two parallel sieves of adequate dimensions. Next we choose a vertical, if possible at a place where the bottom presents a smooth and level rectangle measuring 2 to 4 m in the direction of the stream and 1 to 2 m across. We let then the sieved sand fall into the water through a funnel either in dry condition or in the form of a semi-liquid mixture with water. As soon as the sand grains touch the water, they begin to participate in the movements of the corresponding water particles and describe complicated trajectories, retaining all the time their vertical movement down determined by their hydraulic coarseness u. The first to reach the bottom will be the sand grain which in the whole course of its movement had the vertical compo-

nent directed downwards and equal to its maximal value $w_{max}=(r \sin \gamma)_{max}$ which was added to u. The last one to reach the bottom will be that which had its vertical component the whole time equal to $-w_{max}$.

Inasmuch as we take for granted a symmetric law of probabilities for the variations of the angles β and γ, the main mass of the sand will correspond to the equality $\Sigma+w_{max}=\Sigma-w_{max}$, and will sink to the bottom with the velocity u. Thus, in the first approximation, we obtain for the terms of sinking:

$$T_{pioneer}=\frac{H}{u+w_{max}}; \qquad T_{max.\ concentr.}=\frac{H}{u};$$

$$T_{final}=\frac{H}{u-w_{max}}, \text{ if } u>w_{max}; \quad T_{final}=\infty, \text{ if } u<w_{max};$$

and for the corresponding distances:

$$L_{pioneer}=\frac{Hv}{u+w_{max}}; \qquad L_{max.\ concentr.}=\frac{Hv}{u};$$

$$L_{final}=\frac{Hv}{u-w_{max}}, \text{ if } u>w_{max}; \quad L_{final}=\infty, \text{ if } u<w_{max}.$$

Hence we can determine the mean longitudinal velocity

$$v=\frac{L_{max.\ concentr.}}{T_{max.\ concentr.}}=\frac{L_{max.\ concentr.}\,u}{H}$$

as well as the relative value of the maximal vertical component by means of the following formulae:

$$\Delta L_{rise}=L_{max.\ concentr.}-L_{pioneer}=Hv\left(\frac{1}{u}-\frac{1}{u+w_{max}}\right);$$

$$\Delta L_{fall}=L_{final}-L_{max.\ concentr.}=Hv\left(\frac{1}{u-w_{max}}-\frac{1}{u}\right);$$

$$\frac{\Delta L_{rise}}{\Delta L_{fall}}=\frac{u-w_{max}}{u+w_{max}}=\frac{1-\frac{w_{max}}{u}}{1+\frac{w_{max}}{u}}=f\left(\frac{w_{max}}{u}\right).$$

Thus, the form of the ridge characterizes w_{max}/u and w_{max} itself, since u is known.

The presence of the transversal component of the current is indicated by the dimensions of the slopes of the ridge. The extent of the slope at the point corresponding to the maximum concentration of the sand, i. e. the highest point of the ridge, gives us the possibibity to determine the angle β by means of the formula $L_{trans}=v\ T \sin \beta$.

In order to find the three components of the current it is thus sufficient to know the relative position of five points in the plane with respect to the projection of the place where the sand fell into the water (Fig. 3), namely the four extremal points reached by the sand grains and the point corresponding to the maximum height of the ridge.

The method can be also applied in the case when the ridge extends to infinity. In this case the ratio w_{max}/u can be determined by the character of the ridge profile in the neighbourhood of its turning point.

When the depth is not too great, the required points may be fixed visually or with the help of an optical instrument under adequate light conditions. When the depth exceeds 3 m, the points may be fixed by a diver, which does not present any difficulties.

The method suggested gives us an evaluation of the turbulence along the whole vertical. If, for some reason, it is interesting to obtain a more detailed knowledge, the experiment may be repeated by dropping the sand from a rubber tube plunged into the water to a definite depth. The combined consideration of the ridges corresponding to two positions of the tube enables us to determine the turbulence of a layer between any two levels.

The method may be also used in the case when on the usual turbulence there is superimposed a circulating current directed upwards or downwards. In this case two experiments are required with sands of two different hydraulic sizes.

Finally, if we consider the turbulence of a current not only as a microphenomenon, but also as a macrophenomenon caused to a considerable

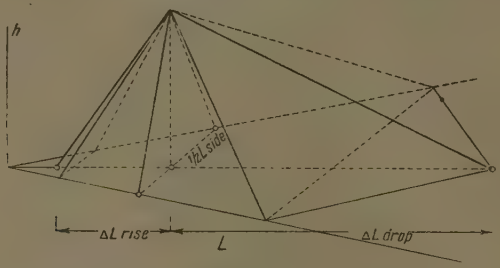

Fig. 3.

extent by circulating undercurrents, the experimental determination of such turbulence may be effected by a turbo-meter designed by the author [3] and based upon the current washing colour out of a cylinder. The ratio of the test concentration after the exposition to the standard colour used for filling the cylinder gives us the component of the velocity, say in the vertical direction, while the ratio of this component to the longitudinal velocity is an approximate characteristic of the turbulence.

Series of experiments made by the author, using the methods described above on different water objects, tended to show that the fundamental relation between the coefficient of turbulent exchange, obtained by calculation from the characteristics given above and the velocity may be represented in the form $A=dv^{2-\delta}$. The value of δ, however, requires a more accurate determination.

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Received
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

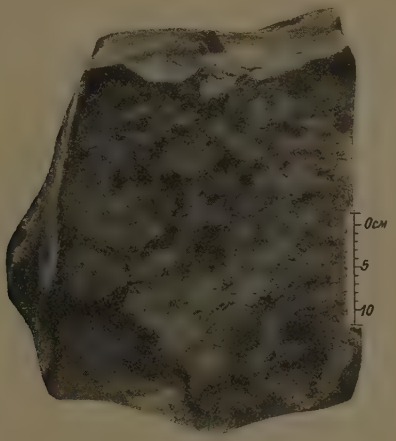
ASTROPHYSICS

THE KAÏNSAZ STONY METEORITE OF 1937

By L. Ş. SELIVANOV

(Communicated by V. I. Vernadsky, Member of the Academy, 15. V. 1938)

The fall of large specimens of a stony meteorite took place on the 13th of September, 1937, at 2.15 p. m. (according to the astronomical time of the



third belt) near the collective farm Kaïnsaz of the Musliumoff district of the Tatar Republic (USSR). A bolide with a short fiery tail passed in flight towards the north-west, leaving behind a smoky trail which remained visible for 10 to 15 min. thereafter; then the fire ball burst into fragments in mid-air. The detonation could be heard at a distance of 130 km.

The fragments of the meteorite were found spread over the surface of an ellipse running in the north-westerly direction, with axes extending respectively over 40 and 7 km.

The largest fragment, well preserved and weighing about 1025 kg (Fig. 1), fell in Kaïnsaz (55°26′ N. lat. and 32°55′ E. long. from Pulkovo), this locality being the extreme NW point of the ellipse, while the smallest one, of the size of a nut, was found in the opposite point of the ellipse—near the village of Kosteevo. Between these two points a number of smaller fragments were found, weighing respectively 53 kg (Kaïnsaz), 27.5 kg (Tash-Elga), 22 kg

(Krasnyi Yar), etc.; all in all, 15 fragments were found with an aggregate weight of more than 200 kg.

The cavities formed in result of the fall of the larger fragments had the appearance of small holes extending, with a single exception, to a depth which was approximately equal to the vertical size of the fragment.

The fragment were in colour dark-grey with a slight tinge of green and were covered with a black and somewhat coarse fusion crust with characteristic piezoglypts. The specimens influenced the magnetic needle to a small extent. The meteorite was preliminarily classed under the group of chondrites.

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Academy of Sciences of the USSR.
Moscow.

Received
23.V.1938.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

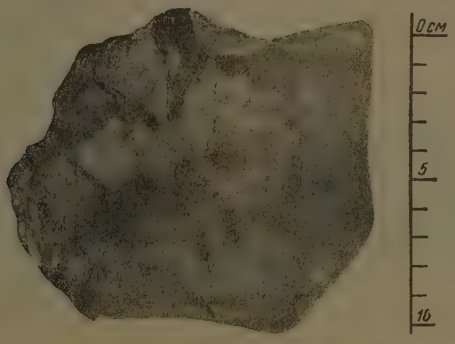
ASTROPHYSICS

THE STONY METEORITE KAPTAL-ARYK OF 1937

By L. P. MALIUGA

(Communicated by V. I. Vernadsky, Member of the Academy, 15. V. 1938)

The stony meteorite described fell on the 12th of May, 1937, at about 8.45 p. m. (by the time of the fourth belt) in the centre of the kishlak (village) Kaptal-Aryk of the Kalinin district of the Kirghiz Republic (USSR), 70 km



west of the town Frunze and 7 km to the south-east of the station Kainda of the Frunze-Aryss railway. The geographical co-ordinates of the spot are: $\varphi = 42°27'$; $\lambda = 43°02'$ from Pulkovo. The flight of a fiery ball was observed, this ball appearing slightly reddish in colour. It was accompanied by large amount of small sparks forming a bright but rapidly vanishing trail after the bolide.

The bolide flew from east to west. All the phenomena of the fall lasted no more than 2 to 3 sec. At first a noise resembling to hum of several aeroplanes was perceived, then came a deafening peel as of thunder which was heared within a range of 20 to 30 km. The flight of the bolide was observed in the town of Frunze, situated at 70 km from the place of fall. The locality surrounding the spot of the fall failed to be brightly lit up; the illumination at a distance of 45 km from the spot, in the Karl Marx collective farm, resembled moonlight. The meteorite penetrated into the hard ground of the village street to a depth of 60 cm.

The surface of the meteorite consisted of a smooth black crust 0.5 mm thick. It contained characteristic piezoglypts. The core of the meteorite is inter-

sected by a network of black veins. Grains of nickeliferous iron were observable on the fracture (Fig. 1). The original weight of the meteorite was set at 3.5 kg.

The meteorite may be preliminarily classed among veined chondrites.

The aggregate weight of the three parts of the meteorite received by the Academy of Sciences of the USSR equals 2904 g.

Meteorite Committee.
Academy of Sciences of the USSR.
Moscow.

Received
23.V.1938.

Comptes Rendus (Doklady) de l'Akadémie des Sciences de l'URSS
1938. Volume XX, № 4

ASTROPHYSICS

THE STONY METEORITE LAVRENTIEVKA OF 1938

By E. L. KRINOV

(Communicated by V. I. Vernadsky, Member of the Academy, 15. V. 1938)

The meteorite fell on the 11th of January, 1938, at about 1.30 p. m. (according to the time of the fourth belt) near the village Lavrentievka of the

Fig. 1.

Fig. 2.

Orenburg region ($\varphi = 52°37'$ and $\lambda = 21°14'$ from Pulkovo). The meteorite was lifted from the ground immediately after its fall by T. A. Zhigunov, a huntsman from the mentioned village, who at the time had been at a distance of 18 to 20 m from the spot. The mentioned witness did not observe any light phenomena, but he distinctly recollects having heard just before the fall deep prolonged sound with three detonations coming from the north, these detonations increasing successively in force and following one another within approximately the same periods of time. Other inhabitants of Lavrentievka and other surrounding villages also heard these detonations, but, just as Zhigunov, none of them observed any light phenomena, though the sky was entirely free of clouds at the moment.

The meteorite fell onto frozen ploughed ground deprived of any snow cover and penetrated the soil to a depth of 5 cm. In falling down, the meteorite caused the clods of frozen ground to fly in different directions within an area of 2 m in radius. T. A. Zhigunov also observed the meteorite before coming to a standstill to rotate about its axis during a certain period of time.

Following two minutes after the fall, the meteorite still remained so hot that it could not be held in the hand, and full ten minutes had to elapse before it was cool enough to be freely handled.

In course of the fall a small fragment became detached from the meteorite. Further on, three more small pieces were broken off from the main mass. These three fragments superimposed upon the main bulk plainly disclose the lens-like shape of the meteorite (Figs 1 and 2). The weight of the fragments gathered is as follows:

Fragment	№ 1	(accession № 1103)	— 135.20 g
»	№ 2	» № 1104	— 77.45 »
»	№ 3	» № 1105	— 58.55 »
»	№ 4	» № 1102	— 510.80 »
»	№ 5	» № 1107	— 11.45 »
Small crumbs and powder		» № 1106	— 0.15 »
		Total	— 793.60 g

Besides this, something like 200 g was probably lost at bursting. All the fragments have been received by the Meteorite Collection of the Academy of Sciences of the USSR.

The meteorite may be classed as a «white» chondrite and contains a rather considerable amount of chondrules and metallic inclusions. The opaque smooth crust of the meteorite is blackish-brown in colour and completely deprived of piezoglypts.

The survey of the locality and of the circumstances attending the fall of the meteorite was effected by the writer.

Meteorite Committee.
Academy of Sciences of the USSR.
Moscow.

Received
23. V. 1938.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

ASTROPHYSICS

THE SPECTRUM REFLECTING POWER OF STONY METEORITES

By E. L. KRINOV

(Communicated by V. I. Vernadsky, Member of the Academy, 24. V. 1938)

Investigations of the spectrum reflecting power of meteorites are of significance for a number of astrophysical problems, and in particular, for making comparison between the reflecting power of a meteorite surface and that of the surface of planets and asteroids(1). Such investigations are

Table 1

Specimen №	Place of recovery	Type	Date of fall	Remarks
196	Staroye Boriskino	Black chondrite	20.IV. 1930	Surface of fracture preserved very well
778	Migey	Black chondrite	18.VI. 1889	
724	Caucasus	Black chondrite	20th century	Surface of fracture slightly oxidized
713	Sungatch	Dark-grey chondrite	10.IV. 1935	Surface of fracture preserved very well
555	Demina	Intermediate chondrite	11.IX. 1911	Fracture strongly contaminated
779	Savtchenskoye	Intermediate chondrite	27.VII. 1894	Surface of fracture preserved very well
716	Saratoff	Intermediate chondrite	IX. 1918	
75	Okhansk	Intermediate chondrite	30.VIII. 1887	
2	Zhigailovka	White chondrite	12.X. 1787	Surface of fracture slightly oxidized
727	Yurtook	Achondrite-amphoterite	2.IV. 1936	Surface of fracture preserved very well
780	Vavilovka	Achondrite-amphoterite	19.VI. 1876	
718	Brient	Achondrite	19.IV. 1933	
749	Staroye Pesianoye	Achondrite	2.X. 1933	

carried out systematically by the Meteorite Committee (Department for Mathematical and Natural Sciences of the Academy of Sciences of the USSR).

In the present case studies were made of 13 stony meteorites from the collection of the Academy of Sciences, these specimens typifying the main classes of stone meteorites. Table 1 gives brief characteristics of all the specimens investigated.

The method of photographic spectrophotometry* was used in the investigation. This method consisted in the following. With the help of a small prism spectrograph reflection spectra from the fracture surfaces of the meteorites were thrown upon sensitive photographic plates. Then on each of these plates a number of spectra from a standard surface were photographed, these spectrograms being one by one gradually weakened in a definite ratio

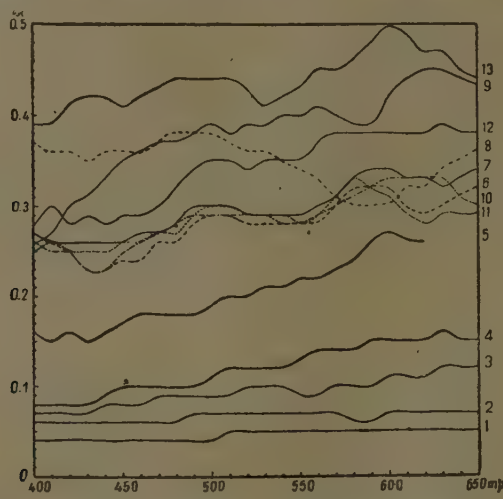


Fig. 1. The figures on the right side signify № of meteorites in Table 1.

by means of a special device attached to the spectrograph. Thus, definite photometer scales appeared side by side with the spectra. To provide for the standard surface, a gypsum plate was taken whose integral reflection coefficient equalled 0.96 (as compared with magnesia).

In spectrographing, both the meteorites and the standard surface were set in a horizontal position and were illuminated by natural daylight (the sky being cloudless). The collimator of the spectrograph was directed towards the surfaces which were to be spectrographed at an angle of 45°, being at the same time at an azimuth of 90° relatively to the Sun. All the spectrographs were obtained with the same exposures on Ilford Soft Gradation Panchrom 6472-A backed plates and were developed under standard conditions with the help of paraamidophenol developer.

The spectrograms obtained were then measured with the help of a recording Moll microphotometer. From the measurement of the spectrograms with subsequent treatment of the results, spectrum-brightness coefficient r_λ were obtained, graphically represented in Fig. 1.

Careful examination of the group in question discloses the following:

* At present a new laboratory apparatus for investigating the spectrophotometric properties of meteorites is being developed.

1. A very great difference in brightness is observable with different types of meteorites. Thus, the brigthest meteorite—Staroye Pesianoye—was found to be ten times brighter than the darkest one, that of Staroye Boriskino (Fig. 2).

2. All the meteorites studied with the exception of a single one, namely the Okhansk meteorite, have their reflecting power gradually rising from the violet to the red end of the spectrum. On the other hand, the enhanced reflectivity of the Okhansk meteorite is observable in the short wave-length region of the spectrum.

3. In comparing the data on the spectrum reflecting power of the meteorites with the petrographic type of the latter, it was clearly manifest that no connection whatsoever existed between these two chracteristics (cf. Yurtook, Vavilovka, Savtchenskoye and Saratoff meteorites).

On the basis of the spectrum brightness coefficients of all the meteorites investigated r_λ, calculations were made of ratio K, i. e. the ratio of the

Fig. 2. *1*—Staroye Boriskino; *2*—Staroye Pesianoye.

integral brightness coefficient of the short wave-length part of the spectrum ($\lambda\lambda = 400—525$ mμ) to the integral brightness coefficienf of the remaining part of the spectrum ($\lambda\lambda = 525—650$ mμ). The obtained values of K are tabulated below (Table 2).

Table 2

	K
1. Okhansk	1.113
2. Staroye Pesianoye	0.927
3. Migey	0.917
4. Vavilovka	0.900
5. Saratoff	0.867
6. Savtchenskoye	0.864
7. Staroye Boriskino	0.848
8. Brient	0.828
9. Zhigailovka	0.824
10. Yurtook	0.813
11. Caucasus	0.794
12. Sungatch	0.768
13. Demina	0.706

From this table it follows that but a single meteorite—the Okhansk one—has a value of K surpassing unity, while with all of them the magnitudes of K tend to decline from meteorite to meteorite, this being as should be noted in full agreement with the impressions obtained from the visual examination of the meteorites.

In comparing the magnitudes of the spectrum brightness coefficients of chondrite meteorites with the respective descriptive characteristics given above such as «white», «intermediate», «dark», and «black», it was found

that these estimations closely approached the spectrum brightness data. On that account all the meteorites investigated with closely related magnitudes of r_λ were classed together under 4 respective classes, the mean values of r_λ being calculated for each group (Table 3).

Table 3

λ, $m\mu$	Classes				Remarks
	I	II	III	IV	
400	0.06	0.12	0.26	0.32	I—corresponds to «black»
410	0.06	0.12	0.26	0.33	
420	0.06	0.12	0.25	0.34	
430	0.06	0.12	0.24	0.34	II—corresponds to «dark»
440	0.06	0.12	0.24	0.35	
450	0.06	0.13	0.25	0.35	III—corresponds to «intermediate»
460	0.06	0.14	0.26	0.36	
470	0.06	0.14	0.27	0.37	
480	0.06	0.14	0.27	0.38	IV—corresponds to «white»
490	0.07	0.14	0.29	0.39	
500	0.07	0.15	0.29	0.39	
510	0.07	0.16	0.29	0.38	
520	0.07	0.16	0.29	0.38	
530	0.07	0.16	0.29	0.38	
540	0.07	0.17	0.28	0.38	
550	0.07	0.17	0.29	0.38	
560	0.07	0.19	0.29	0.39	
570	0.07	0.18	0.30	0.38	
580	0.07	0.19	0.32	0.38	
590	0.07	0.20	0.33	0.39	
600	0.07	0.21	0.32	0.40	
610	0.08	0.21	0.31	0.41	
620	0.08	0.21	0.31	0.41	
630	0.08	0.21	0.31	0.41	
640	0.08	0.20	0.31	0.40	
650	0.08	0.20	0.31	0.41	
Mean r	0.07	0.16	0.29	0.38	

Further, the mean values of r for the whole spectrum were calculated for every class. Thus, class I, or the verbal evaluation «black», finds itself a correspondence in meteorites with a mean brightness coefficient equalling 0.07; class II «dark»—in coefficient 0.16; class III «intermediate»—in coefficient 0.29 and class IV «white»—in coefficient 0.38. It should, however, be emphasized that the stated photometrical classificiation is to be considered as no more than a first approximation.

Meteorite Committee.
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Received
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

PHYSICS

PRODUCTION OF PAIRS IN NITROGEN BY γ-RAYS

By L. V. GROŠEV and I. M. FRANK

(Communicated by S. I. Wawilow, Member of the Academy, 28. VI. 1938)

The process of pair production in light elements by γ-rays remained up to the present almost unstudied. This gave us the idea to investigate pair production in nitrogen using for this purpose the Wilson chamber method. It is well known that for the case of heavy gases the energies of electrons and positrons refuse to be measured with adequate accuracy by the curvatures of their respective tracks. The substitution of heavy gas in the Wilson chamber by light one permits of a considerable improvement of the measurement accuracy. In the case of nitrogen the study of the problems bearing on the particle energy and, in particular on the energy balancing, may yield results of considerably higher significance.

In the present work we had recourse to the same arrangement that had been used by us when observing pairs in krypton (1). As the probability of pair-creation in nitrogen is but very small, a much stronger source of RdTh was made use of (about 140 mg equ.). Similar to the former case, the source was put into a big lead block.

γ-rays, filtered through one centimeter of lead, were penetrating as a narrow beam into the chamber through a window made in the lateral cylinder of the Wilson chamber; this window was covered with a thin copper foil. The chamber filled with air under atmospheric pressure was placed within the magnetic field, the tension of the field being on the average equal to 1750 Gauss. Alcohol vapours were made use of; the main part in pair production was played by nitrogen ($Z=7$), the rôle of oxygen being rather insignificant ($Z=8$). The total of 1680 stereoscopic photographs was obtained. Below are given preliminary data referring to the energies of the pairs produced. Energy measurements were effected for all the pairs whose vertices were lying in the gas within the beam of γ-rays. The results obtained are plotted in Fig. 1. Along the axis of the abscissae is plotted the sum of kinetic energies of a positron and an electron, along the axis of the ordinates—the number of pairs. From this figure may be seen that about the energy of 1600 ekV a strong maximum is observed. It corresponds to the principal radiation line of ThC'', possessing an energy of 2620 ekV. The broadness of this maximum is within measurement error.

It is known from a number of studies (1–3) that in heavy gases instead of the sharp maximum a very broad curve is observed in the neighbourhood of 1600 ekV. Comparing these data with the results obtained for pairs in nitrogen, we arrive at the conclusion that this broadness should be entirely

accounted for by the errors of energy measurements, which are very high in heavy gases, since in this case the paths of electrons and positrons suffer severe distortion due to scattering.

Besides pairs from the principal line of ThC'', there is observed a certain number of pairs possessing considerably smaller energies. However, no identification of these pairs as to their origin from other lines of our source can be made, statistical material being scanty.

To study energy distribution between positron and electron, pairs from the principal line of ThC'' have been isolated. Among these were classed all the pairs with energies from 1450 ekV to 1750 ekV. As may be seen from Fig. 1, the number of such pairs equals 18. The mean energy for these pairs is equal to 1580 ekV, which fairly agrees with the value expected—1600 ekV. It should be noted that in most of these pairs the positron has a considerably higher energy than the electron (in 13 pairs out of 18). The distribution of electrons and positrons according to their respective energies is given in

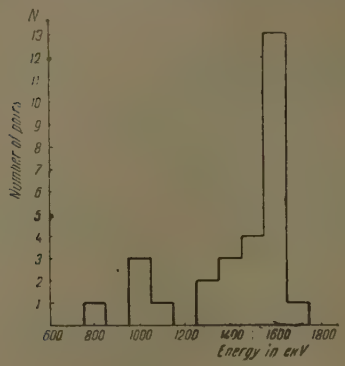

Fig. 1.

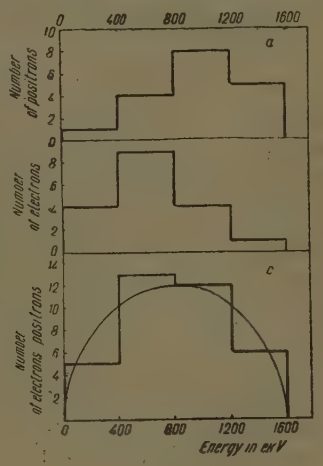

Fig. 2.

Fig. 2. Along the axis of abscissae is plotted the kinetic energy of particles, along the axis of ordinates—their number. Fig. 2, *a* refers to positrons, Fig. 2, *b*—to electrons. As may be seen from the figure, these distributions exhibit a marked asymmetry.

In Born's approximation the distribution for electrons and positrons as to their energies should coincide. The curve of distribution according to energies computed in this approximation by Bethe and Heitler [4] is given in Fig. 2, *c* (for $h\nu = 6\,mc^2$). The total number of distributions for electrons and positrons obtained by us is shown in the same figure. From this figure may be seen that the results obtained from that averaged distribution approach that theoretically expected (in Born's approximation).

If the effect of the nucleus charge be taken into account, then the distributions both for electrons and positrons should display an asymmetry. Owing to this, the mean energy for positrons should be higher than that for electrons. This difference between the mean energies $\bar{E}_+ - \bar{E}_-$ may be expected[4] to change in proportion to Z. For lead the magnitude $\bar{E}_+ - \bar{E}_-$ had been theoretically calculated by Hulme and Haeger [5], and found equal to 270 ekV. Hence, making use of the proportionality Z, this magnitude may be computed for any element. The values of this magnitude found for krypton [6] and iodine [2] agree with those theoretically predicted. For nitrogen the magnitude of $\bar{E}_+ - \bar{E}_-$ ranges about 20 ekV.

If one assumes that the pairs are produced at a distance fron the nucleus not less than $\frac{\hbar}{mc}$, as it usually occurs (4), then we obtain for the difference $\overline{E}_+ - \overline{E}_-$ the maximum value, equal to $\frac{2Ze^2mc}{\hbar}$ which for nitrogen makes up 50 ekV. Hence it follows that in the case of nitrogen the effect of the nucleus is practically negligible. This conclusion, however, conflicts with the data obtained by us for nitrogen. For the 18 pairs from the principal line of ThC″ radiation, the difference between the mean energies of positrons and electrons proves actually to be about 300 ekV. If we take into account all the pairs with energies higher than 1000 ekV (27 pairs), then we obtain for $\overline{E}_+ - \overline{E}_-$ a magnitude of about 200 ekV. The data obtained by us are, however, still not sufficient to maintain with full confidence that this discrepance between experimental and theoretical data is significant. It should be noted that in the course of pair selection those of them might possibly have been omitted in which the whole energy was concentrated in the electron (such pairs being difficult to distinguish from Compton electrons). Owing to this fact, the magnitude of $\overline{E}_+ - \overline{E}_-$ found by us may prove exaggerate. However, the discrepance observed may scarcely be accounted for solely by that circumstance.

In concluding, we should like to note that for the 4 pairs previously obtained by Skobeltzyn (7) in nitrogen, the positron was also carrying a larger energy than the electron. This inconsistency has been pointed to by Zuber (3) in his recent paper. This author has shown that the magnitude of $\overline{E}_+ - \overline{E}_-$ for pairs in argon markedly surpasses the value computed from the data obtained by Hulme and Jaeger.

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Received
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1938. Volume XX, № 4

PHYSICS

ORIENTATION OF SURFACE CRYSTALS OF BRASS BY ABRASION

By N. A. SHISHAKOV and V. I. KASATOCHKIN

(Communicated by W. A. Kistiakowsky, Member of the Academy, 27. V. 1938)

A great deal of cases is known in electron diffraction experiments where the orientation axis of crystals in very thin layers is directed perpendicular to the surface of the underlying solid body. We have observed, however,

Fig, 1.

Fig. 2.

a case where this axis lays in the surface itself. Whilst the surface of a brass abraded without preferred direction affords an electron diffraction pattern of random orientation of crystals (Fig. 1), the pattern from the same brass abraded in some definite direction exhibits almost thorough orientation of a part of the brass crystals at perpendicular incidence of the ray to the scratches. The orientation axis coincides in this case with the [110] direction (Fig. 2). This fact can be easily seen not only from the computation of the angular distribution of the spots, but also by simply measuring the distance between the vertical layer-lines (5.9 mm) which corresponds to the identity period 2.53 Å., the actual distance between (100) planes being in the case of copper 2.55 Å. That the Fig. 2 really represents a rotation photograph, is confirmed also by complete disappearance of spots with the electron ray directed along the scratches, as in Fig. 1. No such effect is observed in the case of uni-directional abrasion of clean copper, iron and steel.

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Received
29. V. 1938.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

PHYSICS

ON THE RECTIFICATION OF CURRENT AT THE BOUNDARY BETWEEN TWO SEMI-CONDUCTORS

By B. DAVYDOV

(Communicated by A. F. Joffe, Member of the Academy, 27.V.1938)

When an electric current passes through the contact between two electronic semi-conductors with electrical conductivities of different types (free electrones and positive «holes») the concentrations of free charges should be altered near the contact.

Let a semi-conductor I have principally «holes» conductivity and semi-conductor II principally normal electronic conductivity so, that the concentrations of free charges in them:

$$n_1' \ll n_2'; \quad n_1'' \gg n_2''. \tag{1}$$

[The subscript $_1$ refers to electrons, and subscript $_2$ to holes; all magnitudes referring to the semi-conductor I will be denoted by a single mark, (′) and those referring to II—by a double one (″).]

If (+) be applied to the semi-conductor I and (—) to II, the holes in the semi-conductor I and the free electrons in II will both move towards the contact, and when the current is steady they must disappear there. Let their respective lives be denoted by τ_1 and τ_2. Then the number of free electrons and holes disappearing per unittime at the expense of those newly formed by thermal movement is $\frac{(n_{1,2}-n_{1,2}^0)}{\tau_{1,2}}$, where $n_{1,2}^0$ denotes their normal concentrations. Consequently, in order to cause free charges disappear close to the contact we should have at this place: $n_{1,2} > n_{1,2}^0$. If, on the other hand, the current flows in the opposite direction, free charges should be formed near to the contact, where we should have: $n_{1,2} < n_{1,2}^0$.

These changes in the concentrations and the changes in the contact potential difference which they cause result in the rectification of the electric current (1).

The general equations for the concentrations $n_{1,2}$ are of the form:

$$\frac{di_{1,2}}{dx} + \frac{n_{1,2}-n_{1,2}^0}{\tau_{1,2}} = 0, \tag{2}$$

where

$$i_1 = -\mu_1\left(En_1 + \frac{kT}{e}\frac{dn_1}{dx}\right), \quad i_2 = \mu_2\left(En_2 - \frac{kT}{e}\frac{dn_2}{dx}\right). \tag{3}$$

$\mu_{1,2}$ denotes the mobilities of the electrons and holes in a field E. The total current

$$i = e(i_2 - i_1) = \text{const.} \tag{4}$$

The boundary conditions for the surface separating the semi-conductors ($x=0$) will be:

$$\frac{n'_{1,2}(0)}{n^{0'}_{1,2}} = \frac{n''_{1,2}(0)}{n^{0''}_{1,2}} \exp\left(\pm \frac{eV_k}{kT}\right). \tag{5}$$

Here V_k denotes the change in the contact potential difference (which is considered as positive, if the energy of electrons on the right is increased). The upper sign relates to the electrons and the lower one to the holes. Moreover, we have the condition of continuity of currents:

$$i'_1(0) = i''_1(0). \tag{6}$$

From (5) we find that for $x=0$:

$$\frac{n'_1 n'_2}{n^{0'}_1 n^{0'}_2} = \frac{n''_1 n''_2}{n^{0''}_1 n^{0''}_2}, \tag{7}$$

$$eV_k = kT \ln\left(\frac{n'_1 n^{0''}_1}{n''_1 n^{0'}_1}\right). \tag{8}$$

In a previous work a strict solution of this problem was given for one particular case which, however, is not actually met with. We may now obtain an approximate solution under the condition that aside from (1) we have:

$$\frac{n'_1}{\tau'_1} \ll \frac{n'_2}{\tau'_2}; \qquad \frac{n''_1}{\tau''_1} \gg \frac{n''_2}{\tau''_2}. \tag{9}$$

It may be thought that these conditions are generally fulfilled.

Denoting

$$\nu_{1,2} = n_{1,2} - n^0_{1,2}, \tag{10}$$

we find from (2) and (4)

$$\frac{\nu_1}{\tau_1} = \frac{\nu_2}{\tau_2}. \tag{11}$$

In the semi-conductor I we shall have $\nu'_1 \sim n^{0'}_1$. Consequently, in view of (1) and (9) $\nu'_2 \ll n^{0'}_2$. Therefore, of the four terms containing E which enter into the expression for $i_{1,2}$ ($En^{0'}_1$, $E\nu'_1$, $En^{0'}_2$ and $E\nu'_2$) we may neglect all except $En^{0'}_2$. In exactly the same way, for semi-conductor II we may reject all but $En^{0''}_1$.

For semi-conductor I we, therefore, find from (3) and (11):

$$\mu_2 e n^0_2 E' = i + kT \frac{\mu_2\tau_2 - \mu_1\tau_1}{\tau_1} \frac{d\nu'_1}{dx}. \tag{12'}$$

Substituting this into (2) we obtain an equation for ν'_1:

$$\mu_1 \frac{kT}{e} \frac{d^2\nu'_1}{dx^2} - \frac{\nu'_1}{\tau_1} = 0, \tag{13'}$$

whence

$$\nu'_1 = a' e^{\alpha' x}, \quad \text{where} \quad \alpha'^2 = \frac{e}{\mu'_1 \tau'_1 kT}. \tag{14'}$$

We rejected the solution giving an increase in ν'_1 with decrease in x, assuming that the thickness of the semi-conductor $\gg \frac{1}{\alpha}$.

In the same manner we find for semi-conductor II:

$$\mu_1 e n^0_1 E'' = i + kT \frac{\mu_2\tau_2 - \mu_1\tau_1}{\tau_2} \frac{d\nu''_2}{dx} \tag{12''}$$

and

$$\nu''_2 = a'' e^{-\alpha'' x}, \quad \text{where} \quad \alpha''^2 = \frac{e}{\mu''_2 \tau''_2 kT}. \tag{14''}$$

We find from (7) neglecting $\frac{\nu_2'}{n_2^{0'}}$ and $\frac{\nu_1''}{n_1^{0''}}$:

$$\frac{a'}{n_1^{0'}} = \frac{a''}{n_2^{0''}}. \tag{15}$$

Finally from (6) and (14) we obtain:

$$\frac{n_1^{0'}}{a'} = \frac{kT}{i}(\alpha'\mu_1' n_1^{0'} + \alpha''\mu_2' n_2^{0''}). \tag{16}$$

We can now calculate the additional «concentrational» potential difference V_c which appears when the current passes through the contact between our semi-conductors. In view of (1) their normal specific resistance will be:

$$\rho' = \frac{1}{e\mu_2' n_2^{0'}}; \qquad \rho'' = \frac{1}{e\mu_1'' n_1^{0''}}. \tag{17}$$

Consequently

$$V_c = \int_{-\infty}^{0} (E' - i\rho')\, dx + \int_{0}^{\infty} (E'' - i\rho'')\, dx + V_k =$$

$$= \frac{i}{\alpha'\mu_1' n_1^{0'} + \alpha''\mu_2'' n_2^{0''}} \left(\rho' n_1 \frac{\mu_2'\tau_2' - \mu_1'\tau_1'}{\tau_1'} + \rho'' n_2^{0''} \frac{\mu_2''\tau_2'' - \mu_1''\tau_1''}{\tau_2''} \right) +$$

$$+ \frac{kT}{e} \ln\left(1 + \frac{i}{i_s}\right), \tag{18}$$

where

$$i_s = e\left(\frac{n_1^{0'}}{\alpha'\tau_1'} + \frac{n_2^{0''}}{\alpha''\tau_2''}\right) \tag{19}$$

is the saturation current; here when $i \to -i_s$, $V_c \to -\infty$. If i_s is small, this gives considerable rectification. The total potential difference applied to the semi-conductor will be $V = i R_0 + V_c$, , where R_0 denotes the sum of the normal ohmic resistances.

The first term of (18) is proportional to i, it is of the order of magnitude of $i\rho/\alpha$. If the thickness of our semi-conductors $\gg 1/\alpha$, it may be neglected in comparison with iR_0 and only V_k remains:

$$V_c \approx V_k = \frac{kT}{e} \ln\left(1 + \frac{i}{i_s}\right). \tag{18a}$$

Here the divergence from Ohm's law is connected only with changes in the contact potential difference in distinction from the particular case considered previously.

We find the additional resistance from (18a):

$$R_c = \frac{dV_c}{di} = \frac{kT}{e(i + i_s)}. \tag{20}$$

We further have a simple relation at $i = 0$:

$$\left[\frac{d \ln R_c}{dV_c}\right]_{i=0} = -\frac{e}{kT}. \tag{21}$$

It should be noticed that when the current is very large, the boundary conditions (5) and (6) lose their applicability. In fact when the current is very large, the contact potential difference changes so much that the lower band of permissible energy levels of semi-conductor I comes into contact with the upper band of II (or vice versa if the current is reversed); the electrons and holes can then recombine and dissociate without hindrance at the divi-

ding surface. Consequently, the expressions found above are applicable only provided that

$$-(\psi_2' + \psi_1'') < eV_k < \psi_1' + \psi_2'', \tag{22}$$

where $\psi_{1,2}'$ and $\psi_{1,2}''$ denote the work functions for emission of electrons and holes from a metal into the semi-conductors I and II respectively. R_c should disappear when the current is large, and then $R = \frac{dV}{di} = R_0$.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

PHYSICS

ON THE THEORY OF SOLID RECTIFIERS

By B. DAVYDOV

(Communicated by A. F. Joffe, Member of the Academy, 27.V. 1938)

The considerations given in our previous paper (3) regarding rectification of current at the boundary between two semi-conductors may explain the action of solid, for instance cuprous oxide rectifiers. Cu_2O possesses a hole conductivity which appears owing to the excess of electro-negative O. On the other hand, owing to the method of preparation, the layer contiguous with the copper electrode contains an excess of Cu, which should lead to normal electronic conductivity. Consequently, there exists in this case a kind of a contact between two semi-conductors with conductivities of different types. This should cause an additional resistance depending on the direction of the current, similar to that obtained before. The resistance should be less when the positive charges (holes) pass from the oxide to the copper electrode, as is observed in practice. (If the semi-conductor has, on the contrary, a normal electronic conductivity and the layer contiguous with the metal electrode has a hole conductivity, then the rectifications should proceed in an opposite direction.)

In cuprous oxide rectifiers we have practically not a sharp boundary, but a gradual transition from hole conductivity to mixed and then to normal electronic conductivity. Strictly speaking, equation (2) [loc. cit.(3)] should be solved for $n_{1,2}^0$ depending on x:

$$n_1^0 = n_0 \exp\left(\frac{\psi(x)}{kT}\right); \quad n_2^0 = n_0 \exp\left(-\frac{\psi(x)}{kT}\right), \tag{23}$$

where

$$\psi(x) < 0 \text{ for } x < 0; \quad \psi(x) > 0 \text{ for } x > 0.$$

Here, however, we encounter considerable mathematical difficulties.

The numerical integration carried out by P. V. Melentiev (which will be published separately) showed that the dependence between V and i is fairly well expressed by a formula analogous to (18):

$$V = iR_0 + \left(a\frac{kT}{e} - biR_1\right) \ln\left(1 + \frac{i}{i_s}\right). \tag{24}$$

Here a and b are numerical coefficients ($a \approx 1$, $b \approx 1/4$). R_1 denotes the ohmic resistance of the layer in which the transition from hole conductivity to normal electronic conductivity takes place. Saturation current

$$i_s \sim \frac{n_0}{\beta t}, \quad \text{where } \beta = \left[\frac{d \ln \psi}{dx}\right]_{x=0}. \tag{25}$$

$\frac{1}{\beta}$ is, therefore, the thickness of the transitional layer. (25) is applicable only in the case when β is not too great, and for sharp boundary ($\beta = \infty$) equation (19) must be applied. It should be noticed that if the thickness of the transitional layer $\gg \frac{1}{\alpha}$, the effect should disappear since $\frac{di_{1,2}}{dx}$ may then be neglected in (2).

A comparison with the well-known experimental data for cuprous oxide rectifiers shows that relationship (21) is satisfactorily fulfilled in so far as the order of magnitude is concerned.

In a case of continuous transition from one type of conductivity to the other, the additional resistance R_c should not disappear when the current is large, since the bands of permissible levels cannot here come into contact with one another, as happens in the case of a sharp boundary.

When the current is large, strong fields are formed in the transitional layer. The actual normal concentrations $n^0_{1,2}$ should begin to grow with the increase of the field (increase of conductivity in strong fields). According to (19), the saturation current i_s should also begin to increase as the applied (negative) potential difference increases. The additional resistance R_c attains a maximum and then drops again. The maximum value of R_c should lie in fields of the order approaching that of the breakdown value in the transitional layer. As far as one can judge, this is in agreement with experiment.

Joffe and Frenkel, and others [1] brought forward a theory which explained the action of a cuprous oxide rectifier by a tunnel effect of electrons through a blocking layer. However, it later turned out that cuprous oxide has hole conductivity. In this case the theory of the tunnel effect gives rectification with a sign opposite to that actually observed. Moreover, measurements seem to show that the blocking layer is too thick for the tunnel effect to pass through it.

Besides, it is impossible to explain the action of a cuprous oxide rectifier by cold emission in a blocking layer [2], since in place of (21) this gives $\left[\frac{dR}{dV}\right]_{i=0} = 0$, which does not agree with experiment.

We will now return to the conductions of the passage of current through a sharp boundary between two semi-conductors. Above we assumed that semi-conductor I contains both electrons and holes, and that semi-conductor II similarly contains holes aside from free electrons. It is, however, possible that there will be neither free electrons in I, nor holes in II at all:

$$n'_1 = n''_2 = 0. \tag{26}$$

In this case according to (19) $i_s = 0$ and, consequently, $R_c = \infty$. However, in reality it is necessary to take here into account the direct recombination and dissociation of holes of semi-conductor I and free electrons of II at the boundary between them. (If either n'_1 or $n''_2 \neq 0$, these processes may be neglected.)

In condition (26) it is obvious that $i'_1 = i''_2 = 0$. Then in view of (2) and (4)

$$n'_2 = n^{0\prime}_2, \quad n''_1 = n^{0\prime\prime}_1. \tag{27}$$

Let $\lambda n'_2 n''_1$ electrons and holes recombine on the boundary. The relationship between λ and V_k may be neglected as a first approximation. In the state of thermal equilibrium, i. e. when $V_k = 0$, the numbers of dissociating electrons and holes should be the same. However, dissociation takes place at the expense of thermal energy and the number of dissociating electrons and holes should consequently depend exponentially on V_k; it will be $\lambda n^{0\prime}_2 n^{0\prime\prime}_1 \exp\left(-\frac{eV_k}{kT}\right)$.

The value of the current will thus be:

$$i = e\lambda n_2^{0'} n_1^{0''} \left[1 - \exp\left(-\frac{eV_k}{kT}\right) \right]. \qquad (28)$$

We have obtained a rectification similar to that previously obtained, but in an opposite direction. It is to be expected that condition (26) will be fulfilled rather in insulators.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

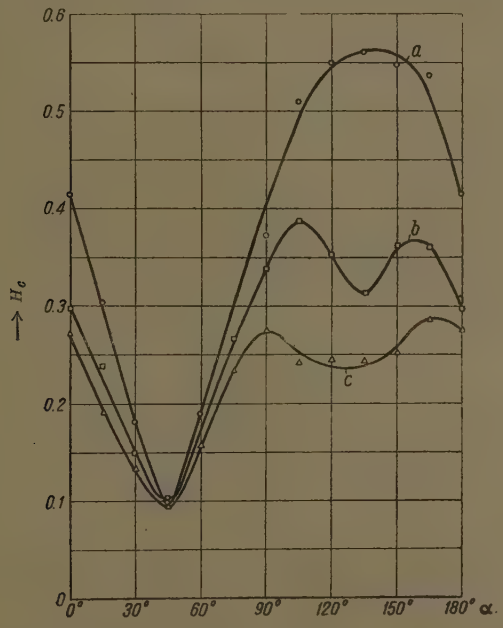
PHYSICS

INFLUENCE OF HEAT TREATMENT IN A MAGNETIC FIELD UPON THE MAGNETIC PROPERTIES OF FERROMAGNETIC SINGLE CRYSTALS

By R. JAANUS and J. SCHUR

(Communicated by S. I. Wawilow, Member of the Academy, 5. VI. 1938)

Using of the previously described method (1) single crystal discs, 20—25 mm in diameter, were prepared of silicon steel with 3.5% Si. These samples were next heated up to 800° and then cooled either under magnetic cover or in a magnetic field applied in various directions in the plane of the disc. After each such treatment the coercive force was measured along various directions, in the plane of the disc too. The directions were fixed by means of angle α, measured from an arbitrary disc diameter used as reference line.



The dependence of the coercive force of the specimens upon orientation, as observed and plotted in Fig. 1 against α, should be regarded as typical. Curve *a* was obtained upon cooling under magnetic cover. Curves *b* and *c* were obtained after cooling in a magnetic field directed at angle $\alpha=135°$ and $\alpha=0°$ respectively. Cooling in the field, applied in the direction $\alpha=45°$, gave again curve *a*.

The character of the curves obtained is in direct dependence upon the orientation of the crystal. A field of the order of several oersteds already calls forth a marked effect; wherefore it can be inferred that the results of Ruder [2] and Sixtus [3] were implicitly affected by the heat treatment in magnetic field from the electric furnace heating coils, used by these authors in course of obtaining and heat treating their samples.

A detailed account of the results of our work, as well as a theoretical interpretation thereof will, be published in the nearest future.

The Ural Physico-Technical Institute.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

PHYSICS

ON THE ISOMERISM OF THE NUCLEI OF URANIUM Z AND URANIUM X_2

By N. N. DMITRIEV

(Communicated by V. I. Vernadsky, Member of the Academy, 31.V.1938)

In a recent paper, Feather and Bretscher (1) in summing up their experimental data on the γ- and β-radiation of uranium Z, advanced the suggestion that uranium X_2 is the metastable state of uranium Z. It would appear from their data that uranium Z is formed only from uranium X_2 through the emission by the latter of γ-rays, and not owing to β-decay of uranium X_1, as had formerly been supposed.

Using their own experimental data as to γ- and β-radiation of uranium Z and those of other writers on the γ-and β-radiation of uranium X_1 and uranium X_2, Feather and Bretscher have plotted a scheme of levels for these nuclei. The scheme is shown in Fig. 1. In that scheme the figures at the right show the energy value of the level in meV in relation to the ground level of U_{11}, which is arbitrarily assumed as the zero level.

The notations β and γ indicate the kind of radiation (electrons and γ-rays respectively). The subscripts of β and γ give the energy value of the radiation, and the superscripts, its relative intensity. The scale is different on both sides of the AA' line. The figures to the left of the level lines indicate the spin values corresponding to these levels.

On the ground of their scheme, those authors came to the conclusion that uranium Z must emit two groups of γ-rays. The experimental study of γ-rays made by the above authors shows that uranium Z does practically emit two groups of γ-rays, this being a direct experimental evidence of the validity of this part of the scheme.

So far as isomerism is concerned, the greatest interest of the scheme shown in Fig. 1 lies in the value of the energy of excitation ε and in the difference of spin values between the level of uranium Z and the ground level of uranium X_2. As to the value of the energy of excitation, Feather and Bretscher were unable to determine it (therefore it is denoted simply by ε in their scheme).

In plotting the above scheme and, more particularly, in determining the spin values for various levels, the above authors made use of Sargent's curves. They also state that, in some respects, there are many different ways of plotting that scheme which, sure enough, affects to a certain point the significance of the conclusions drawn therefrom.

We shall now show that, using the formula given by Bethe (2) in accordance with Weizäcker's theory, one may, without having recourse to Sargent's curves, determine quite unambiguously both difference l of spin values and the value of the energy of excitation ε.

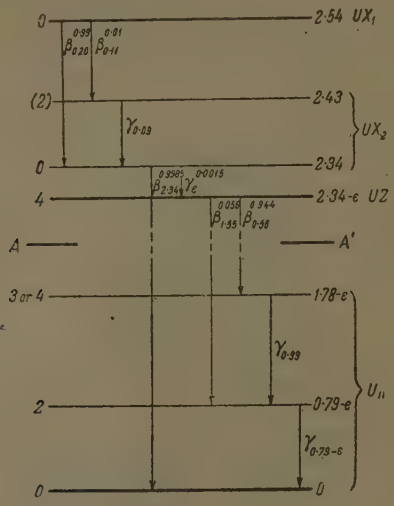

Fig. 1.

Uranium Z is of the type of nuclei possessing an odd atomic number and an even atomic weight ($Z=91$, $A=234$). For other radioactive nuclei of this type, the energy of excitation E_1 for the first excited level has been determined experimentally. The value of that energy given by Feather and Bretscher is shown in the table.

Nucleus	RaC	ThC	RaE	RaC″	ThC″
Z	83	83	83	81	81
A	214	212	210	210	208
E_1 in keV	52.9	238	62	62	40

Besides the elements shown in the table, this type comprises also $MsTh_2$ whose E_1 has not been determined by a direct method, but whose E_1 may be supposed to be small too, according to data available on β-decay. As may be seen from the table, energy E_1 varies within a narrow range, its mean value being about 50 keV. for all the elements so far studied (except ThC). It is therefore natural to assume that value $\varepsilon=E_1$ will lie near 50 keV also for uranium Z.

The lifetime τ_γ of the ground state of UX_2 may be expressed by the formula (2):

$$\tau_\gamma = 5\cdot 10^{-21} l^2 \left(\frac{20}{\varepsilon}\right)^{2l+1} \text{sec.} \qquad (1)$$

Here ε is to be expressed in meV. Feather and Bretscher determined experimentally the period T_γ, corresponding to this γ-radiation. T_γ proved to be $T_\gamma = 12.6$ hours. Hence the mean lifetime τ_γ is $\tau_\gamma = 17.4$ hours $= 6.2\cdot 10^4$ sec. Substituting this value into (1), we obtain for $\varepsilon=50$ keV:

$$1.24\cdot 10^{25} = l^2 (400)^{2l+1}. \qquad (2)$$

If $l=4$, the right side of this expression is about 10 times as great as the left one. If $l=5$, the right side is 10^7 times as great as the left one, and if $l=3$, it is 10^5 times less than the left side. Hence we conclude that, out of all possible values of l, only $l=4$ will satisfy a value of ε equal to about 50 keV. Our discussion of the difference of spin values thus results singlevaluedly in $l=4$ which is in agreement with the scheme shown in Fig. 1. We now can find the exact value of ε (instead of the mean value adopted at the beginning). For this purpose, we substitute into (1) $\tau_\gamma = 6.2 \cdot 10^4$ sec. and $l=4$. Then we obtain:

$$1.24 \cdot 10^{25} = 4!^2 \left(\frac{20}{\varepsilon}\right)^9. \qquad (3)$$

Hence the exact value of the excitation energy $\varepsilon = 51.2$ keV,

We may state in conclusion that, since, as may be seen from (1), τ_γ depends on l to a much greater extent than it depends on ε, we need not, in computing the value of l, assume ε to be equal to the mean value (50 keV).

In fact, as shown by formula (1), we obtain, for any value of ε lying within the limits of E_1, as occurs in reality for nuclei of the type of uranium Z (except ThC, cf. the table), $l=4$, assuming $\tau_\gamma = 17.4$ hours.

We have, thus, found quite unambiguously from the formula of the lifetime of the metastable state both the value of the excitation energy difference of spin values between the level of uranium Z and the ground level of uranium X_2.

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1938. Volume XX, № 4

PHYSICAL CHEMISTRY

ON THE INFLUENCE OF STIRRING ON CURRENT VOLTAGE CURVES OBTAINED WITH A MERCURY CATHODE

By S. JOFA and A. FRUMKIN, Member of the Academy

As was shown elsewhere (1), the maxima on current voltage curves obtained with mercury cathodes are caused by the motion of the mercury surface. This motion, which is especially pronounced with certain polarizations, secures an extra supply of the reducible matter to the cathode, necessary for the maintenance of the maximum current. The parts of the current voltage curves where a limiting diffusion current is observed correspond to a calm state of the cathode. If this explanation be correct, the maxima must

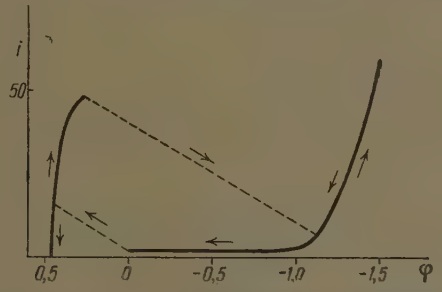

Fig. 1. Current voltage curve with amalgamated platinum cathode in 0.003 norm. $Hg_2(NO_3)_2$+0.003 norm. HNO_3. Arrows indicate in what direction the applied voltage is changed. Potentials referred to normal calomel electrode.

disappear in case a strong artificial stirring is applied to the solution. The realization of a strong stirring with a steady cathode presents certain difficulties, since the mercury is thrown out of the enclosing vessel if the motion of the liquid is sufficiently violent. We have overcome this difficulty by using two methods.

1. Amalgamated Platinum. A thick strongly amalgamated platinum wire was used as a cathode. If the mercury layer is sufficiently thick, a typical current voltage curve with a maximum and a hysteresis loop can be obtained with a cathode of this kind in $Hg_2(NO_3)_2$+HNO_3 solutions, as shown by Fig. 1. The diffusion current is very small compared with the maximum current. With the increase of the HNO_3 concentration, the maximum current decreases (the table). If now a strong jet of the solution is directed onto the electrode, the shape of the curve changes completely

(Fig. 2). The jet escapes from an orifice with a diameter of 1 mm situated in the solution close to the electrode, under a pressure of 120 cm of water, and causes a vigourous stirring of the solution. Under the influence of this stirring, both the maximum and the diffusion current increase, the increase of the former being by far smaller. The difference between the maximum current and the diffusion current, as well as the hysteresis loop, nearly disappear.

	0.003 norm. $Hg_2(NO_3)_2$+HNO_3, saturated with hydrogen					
C_{HNO_3}	0.003	0.006	0.012	0.024	0.048	0.096
i without stirring	45	35	25	16	9	4
i in stirred solution . . .	78	78	73	70	69	70
i diffusion	76	75	68	66	62	63

As shown by the table, the suppressing effect of the foreign electrolyte on the maximum current is no more observed in a stirred solution. Other experiments have shown that the maximum current under these conditions

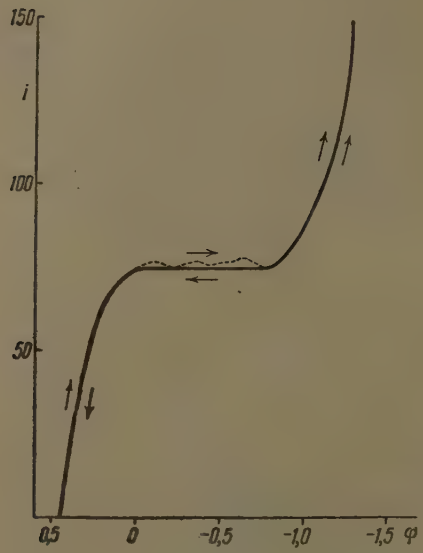

Fig. 2. Current voltage curve with amalgamated platinum cathode in a streaming electrolyte of the same composition.

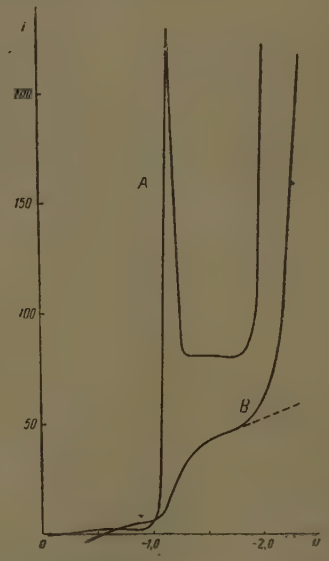

Fig. 3. Current voltage curves of 0.005 norm. Ni $(NO_3)_2$+0.05 norm. KNO_3. *A*—dropping electrode, *B*—dropping electrode in streaming electrolytes.

is proportional to the concentration of mercury. The cathode behaves now in all respects similarly to an ordinary solid electrode. The decrease of the hydrogen overvoltage due to stirring of the solution has, as shown by Fig. 2, a secondary reason: the strong jet of the solution partly bares the electrode from mercury and the catalytic influence of platinum becomes more pronounced.

2. Dropping Electrode. If a similar jet of liquid is directed against the end of the capillary of an ordinary dropping electrode (time of drop formation 1 sec.), the dropping mercury is pulverized and is torn of

from the mercury surface in the form of fine droplets. At the end of the capillary there remains a flat mercury surface which can be used as a cathode. It is obvious that very thorough stirring becomes realized in this way. We shall call this electrode «dropping electrode in streaming electrolyte», although there is no regular drop formation in this case. We have investigated with such electrodes the deposition of both Ni- and Hg_2-ions in solutions saturated with hydrogen. As was shown by Emelianova and Heyrovsky (2), nickel salts give very well developed maxima on current voltage curves.

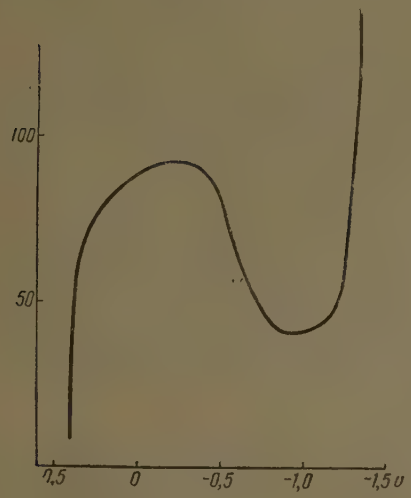

Fig. 4. Current voltage curve of 0.0025 norm. $Hg_2(NO_3)_2 + 0.1$ norm. HNO_3 with ordinary dropping electrode.

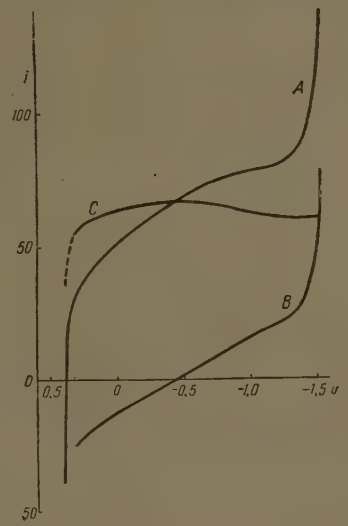

Fig. 5. *A*—current voltage curve of the same solution with a dropping electrode in streaming electrolyte, *B*—similar curve in 0.1 norm. HNO_3, *C*—curve *A* corrected for non-faradayic current and depolarization by HNO_3.

A curve of this kind obtained under ordinary conditions of dropping is given in Fig. 3, *A*. The same solution under conditions of strong stirring gives a curve without a maximum (Fig. 3, *B*) which differs from a curve as obtained with a solid cathode only by a certain slope of the part corresponding to the limiting diffusion current of the nickel deposition and at the beginning of the curve, which should be horizontal. As will be shown, this is caused by the non-faradayic current which is very large in this case, since a large number of small droplets are formed. It is interesting to compare the respective influence of Al-ions on curves *A* and *B*. As has already been shown by Heyrovsky and collaborators, an addition of a small quantity (5.10^{-4} norm.) of an aluminium salt totally suppresses the nickel maximum on curve *A*, while it has but a small influence on curve *B*, slightly increasing its slope in the part corresponding to the limiting current of the Ni-deposition.

In a 0.0025 norm. $Hg_2(NO_3)_2$+0.1 norm. HNO_3 solution with an ordinary dropping electrode a current voltage curve with a well pronounced rounded maximum is obtained (Fig. 4). In a streaming electrolyte (Fig. 5, *A*) this maximum disappears. The part of the curve corresponding to the limiting current of the $Hg_2^{\cdot\cdot}$ deposition has again a marked slope, and a negative current with small voltages is observed. To make clear the reason of these phenomena, we carried out measurements with 0.1 norm. HNO_3 in absence

of mercury salts (Fig. 5, *B*). In this case no electrolytic process is possible until a high cathodic polarization is reached and the whole current up to —1.3 V is a non-faradayic current which is spent on charging the mercury droplets formed. In fact, this current changes its sign from a negative to a positive one, as it should be expected at the electrocapillary maximum—0.48 V. A comparison of curves *A* and *B* shows that our supposition as to the nature of the slope of the middle part of curve *A* was well founded. If we substract from curve *A* the values given by curve *B*, we obtain curve *C* which represents the part of the current caused exclusively by the deposition of mercury ions. The anomalies of curve *A* discussed above have now disappeared, but a faintly pronounced maximum, evidently a rest of the maximum shown in Fig. 4, becomes visible.

The experiments described in this paper show conclusively that the maxima on current voltage curves obtained with mercury cathodes disappear, if the solution is subjected to vigourous stirring, the influence of auto-stirring caused by the motion of the mercury surface becoming under these conditions negligible.

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Received
20. VII. 1938.

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PHYSICAL CHEMISTRY

SYNTHESIS OF NITRIC OXIDE IN A TORCH DISCHARGE. II

By K. N. MOTCHALOV

(Communicated by N. N. Semenov, Member of the Academy, 11. V. 1938)

1. Effect of Pressure. In a previous article (1) a description was given of the dependence of NO yield per kWh in a torch discharge upon the pressure of the reaction mixture N_2 and O_2. For pressure within the range of 740 to 200 mm this dependence proved to be linear.

As will be seen from the newly obtained data given below, the character of the dependence holds also for pressures increased up to 3 atmospheres inclusive.

Table 1

Pressure of reaction mixture in atm .	1	1. 5	2	2. 5	3
Energy yield in g HNO_3 per 1 kWh . .	38. 9	44. 1	51. 6	58. 0	67.3
Same in moles per 1 kWh	0.62	0.70	0.82	0.92	1.07
Concentration NO in volume %	0.94	0.77	0.61	0.57	0.52
Same in mg per 1 l.	11.85	14.56	15.38	17.97	19.67

Dicharge power 270—280 arbitrary units. Duration of experiments 2 minutes. The volume concentration attained during the same period drops, while the weight concentration increases, the discharge power being invariable.

The above-mentioned results were obtained with dried air on a platinum electrode, the methods and the experimental procedure being on the whole the same as in previous work. Some modifications were, however, introduced in the design of the apparatus for work under increased pressures (substitution of a metallic pressure gauge for a mercury one, etc.).

It is worth noting that it was only with difficulty that a spontaneous «lighting» of the discharge could be obtained. For this purpose it was necessary to raise considerably at the initial moment the voltage supplied to the anode of the generator lamp. Besides this, the torch itself, usually resembling a gas flame, was now turned into several small blue tongues which issued from the tip of the electrode.

The considerations advanced in the foregoing article with respect to the mechanism of the influence of pressure upon the yield of NO per un. pow. cons. have found additional evidence in the newly obtained data.

2. Composition of the Reaction Mixture. In order to ascertain the effect upon yield and concentration of the ratio between the amounts of O_2 and N_2 in the reactoin mixture, the reaction vessel was exhausted to the fixed pressure, whereupon pure oxygen was admitted into it. The results of experiments with mixtures so prepared are represented in Table 2.

Table 2

Composition of mixture		Yield per unit pow. cons. in g of HNO_3 per 1 kWh	Volume concentration of NO in %
% N_2	% O_2		
79.1	20.9	30.7	1.5
75	35	39.6	1.8
50	50	43.4	2.0
20	80	35.9	1.7

Duration of experiments—2 minutes.

The maximum effect is produced by an equimolecular mixture corresponding to the stochiometry of the equation

$$N_2 + O_2 = 2NO.$$

A decrease in the content of anyone of the components in the mixture involves a decrease in yield and concentration. Yet, the relative importance of oxygen and nitrogen is far from being the same: a mixture of 80% O_2 and 20% N_2 gives a higher yield than a 80% N_2 and 20% O_2 mixture. These results

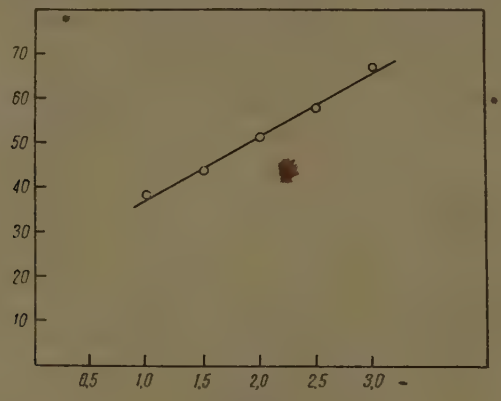

Fig. 1.

are found to be fairly similar to those of Haber and Koenig ([2]) obtained for a voltaic arc. Accordingly, if on the basis of these data as well as the data of Table 2, we are to plot curves «composition of mixture—% of NO», then, as shown in Fig. 2 and 3, they will be found to coincide rather well.

On the basis of the difference between the volumetrically expressed NO yields obtained during the same periods of time from mixtures of 20% N_2 — 80% O_2 and 20% O_2—80% N_2 we can draw the conclusion that the law of the dependence of reaction velocity upon concentration of reactants (law of mass action) is here infringed. It stands to reason that this is caused by the electrical conditions that are peculiar to a torch and mark it out as different from an ordinary high temperature zone.

3. Influence of Electrode Material. In comparing the action of a torch discharge with that of an arc discharge, it would seem in very first place relevant to say that a number of investigators, such as Scheuer ([3]), Brode ([4]) and others, either failed altogether to notice the influence of electrode metal upon the synthesis of NO in the arc or else detected a very insignificant effect only. Later on, Briner ([5]) and his co-workers, while using

alloys of copper with lithium and calcium, obtained a substantial increase in concentration and yield of NO. Coating the electrodes with oxides of alkaline earth metals was also proved by a number of investigators to increase the yield. A definite mechanism was brought to light here: the highest yield was produced by that particular oxide whose ionization potential was lower.

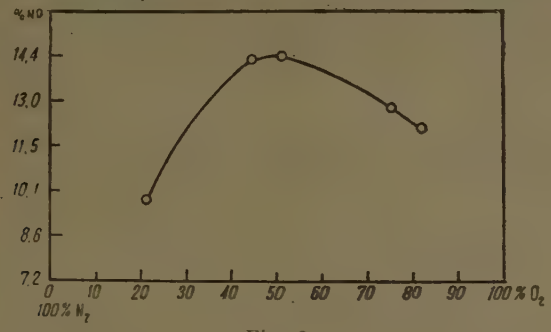

Fig. 2.

The experiments with the torch were effected in the following way: small rods of different metals of equal cross-section and length were prepared, these rods being pointed at one end. The rods were festened as tips to the main electrode (made of copper) and the discharge was made to play

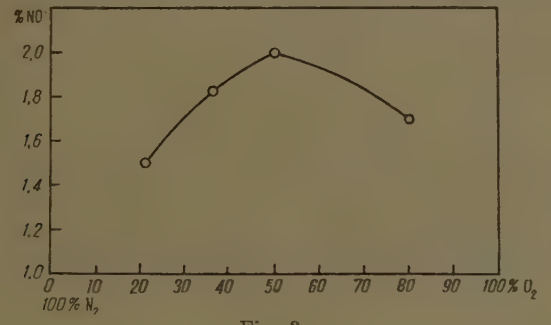

Fig. 3.

upon them. As it proved to be impossible to maintain exactly the same current supply from the generator in course of all the experiments, two series of observations were made, the results of which are represented in Tables 3 and 4.

Table 3

Influence of Electrode Material upon Formation of NO
Series I. Duration of experiments 2 minutes

Electrode metal	Al	Mg	Fe	Ta	W
Yield in g of HNO_3 per 1 kWh . .	47.9	38.9	35.8	37.4	33.7
% NO by volume	1.0	0.8	1.0	1.3	0.8

Table 4

Influence of Electrode Material upon Formation of NO
Series II. Duration of experiments 2 minutes

Electrode metal	Al	Mg	Cu	Pt	W	Fe	Ag
Yield in g of HNO_3 per 1 kWh	39.6	39.6	36.7	36.7	35.9	35.2	34.4
% NO by volume	1.2	1.0	1.1	1.5	0.63	1.3	1.1

The results failed to be altogether distinct.

From both series a clearly shaped mechanism can be deduced only for four metals:

	Al	Mg	Fe	W
Yield per 1 kWh	43.8	39.3	36.8	34.8
% NO by volume	1.1	0.9	1.2	0.7

If we are to attempt to embrace the whole range of the investigated metals in search of a general law, be it only a qualitative one, we can subdivide all of them into groups arranged in a series of a gradually diminished yield:

Al, Mg Fe, Cu W, Ta, Pt Ag

If in order to explain the above results we compare them with the magnitudes of the work spent on tearing away electrons from the given metal, we see these magnitudes growing in inverse proportion to the yield.

Table 5

Element	Work spent on electron emission, eV	Yield in g of HNO_3 per 1 kWh
Al	3.1	Maximum
Mg	3.7	
Ag	3.9	
Fe	4.5	Medium
Cu	4.6	
W	4.7	Minimum
Pt	4.8	

Al and Ag stand out as extreme cases. The small work of electron emission from Al is also probably responsible for its analogous behaviour in other cases, e. g. in a smouldering discharge, where the liberation of ozone on it is comparatively very great and is considered by some authors [7] anomalous.

As regards silver, it may be surmised that the too small yield on this metal, failing to correspond to the work of electron emission (3.9 eV), is related to its very high propensity to become pulverized. This fact was mentioned in our preceding work and is also shown in Table 6, where the metals are given in declining order of their tendency towards pulverization and where Ag stands out as the highest term.

Table 6 [8]

1. Silver
2. Gold
3. Platinum
4. Palladium
5. Magnesium
6. Aluminium
7. Copper
8. Tungsten
9. Tantalum

4. Significance of Torch Size. In comparing the yields in different series of experiments, we see them to differ under seemingly equal conditions. It was noticed that such discrepancies result from differences in the size of the torch. In order to ascertain this dependence more closely, comparative experiments with a large and a small torch were set up. For this purpose, while leaving all the parameters of the apparatus unchanged, only anode voltage was lowered, which offered the possibility of reducing the size of the torch from 7 to 2 cm. Along with the size of the torch its power decreased also, as can be seen from Table 7.

Table 7

Duration of experiments 2 minutes. Platinum electrode

Size of torch in cm	Power of torch in W	Yield in g of HNO_3 per 1 kWh	% NO by volume
7	114	30.3	1.5
2	53	20.2	0.5

In closing this article it should be recalled that in all experiments of the present and the preceding work the generator was working on a semi-period only. The feeding of the lamp anode by current rectified with the help of a mercury rectifier strongly increased both the power and the size of the torch. In this case a torch of up to 30 cm in height and 7—8 cm in diameter (in the upper portion) was readily obtained. Yet, the character of all the relations pertaining to the synthesis of NO remained virtually the same.

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Received
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1938. Volume XX, № 4

CHEMISTRY

ON THE MECHANISM OF THE DISTRIBUTION BETWEEN LIQUID AND SOLID PHASES

By A. POLESSITSKY

(Communicated by V. I. Vernadsky, Member of the Academy, 20. V. 1938)

The equilibrial distribution of small quantities of ions between the liquid and solid isomorphous phases, expressed by Berthelot-Nernst's law, can be attained after various methods (1).

The mechanism of such a process is clear in the case of manifold recrystallization, when each layer of the crystal repeatedly comes into contact with the solution, but it is not clear in the case of crystallization from supersaturated solutions, whether the latter is brought about rapidly, by stirring vigorously, or slowly, without stirring. In this case each layer of the crystal grows from a solution with a different concentration of the distributing substance, and thus the distribution should obey the none-equilibrial logarithmic law (2).

The equilibrium attained in the latter case was explained by the hypothesis that the first stage of crystallization from the supersaturated solutions is the formation of a great number of submicrocrystals throughout the whole of the solution. These microcrystals, which contain the distributing substance in due quantity, grow together, forming visible crystals (3).

If the crystallization is slow, the microcrystals are able to recrystallize many times; the distribution of the microcomponent in them becomes uniform and the visible crystals are formed by the uniform crystals growing together, which results in an equilibrial distribution.

If the crystallization is rapid, owing to vigorous stirring, the latter greatly accelerates recrystallization, momentarily equalizes the concentration in the liquid phase, which results in the growing together of uniform microcrystals and, consequently, in an equilibrial distribution.

In order to check this hypothesis I conducted the following experiments: 1. 2 g of solid $Pb(NO_3)_2$ was added to 25 cm³ of a $Pb(NO_3)_2$ solution, saturated at 0° C. This salt was dissolved by heating the solution, and then crystallized in a thermostat at 0° C by stirring vigorously for 5—6 hours. Owing to the tendency of $Pb(NO_3)_2$ to form supersaturated solutions, crystallization set in already after the solution had been stirred only 20—30 minutes. After stirring for a certain length of time, which varied from experiment to experiment, 1 cm³ of a saturated solution of $Pb(NO_3)_2$ containing Ra was added. In some experiments this solution was added before crystallization set in, in others, after it had set in. The results of the Ra distribution are

given in Table 1 and in Fig. 1. The latter show that if Ra is added only 4 minutes after crystallization has set in, the non-equilibrial distribution coefficient is too small. If, on the other hand, Ra is added 1 minute before crystallization has set in, an equilibrial distribution is still attained. These results are in accordance with our hypothesis.

Table 1

Time between the beginning of crystallization and the addition of Ra in min.	D
−15	3.10
−11	3.12
− 4	3.10
− 1	3.05
+ 4	1.68
+15	1.33
+45	0.22

Table 2

Time between the beginning of crystallization and the addition of Ra in hours	D
−4	2.84
−2	2.12
−0.5	1.10
0	1.74
+2	1.34
+2.3	1.05

These experiments were repeated without stirring, as a result of which all the processes went on much more slowly. The first visible crystals appeared only 3 hours after the beginning of the experiment. The entire crystal-

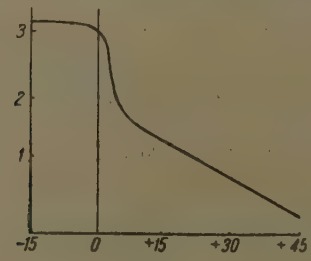

Fig. 1.

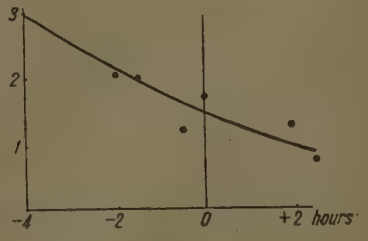

Fig. 2.

lization took about 30 hours. The results of these experiments are given in Table 2 and Fig. 2. In these experiments the addition of Ra 2 hours before the first visible crystals appear, gives a non-equilibrial distribution. Ra cannot penetrate into the microcrystals for lack of time, since recrystallization without stirring goes on too slowly. All these results are in accordance with the hypothesis that equilibrium is attained during the recrystallization of microcrystals, before the appearance of the visible ones.

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Received
26. V. 1938.

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1938. Volume XX, № 4

CHEMISTRY

ON THE EQUILIBRIAL DISTRIBUTION BETWEEN SOLID AND LIQUID PHASES

By A. POLESSITSKY

(Communicated by V. I. Vernadsky, Member of the Academy, 20. V. 1938)

A number of investigations have shown that traces of any radioelement between solutions and isomorphous crystals obtained by crystallizing them from a supersaturated solution, are distributed according to Berthelot-Nernst's law (1).

Recrystallizing the macrocomponent in a saturated mixed solution and recrystallizing mixed crystals in a saturated solution of the macrocomponent gave the same results (2).

On the other hand, if the mixed solution is slowly evaporated at a constant temperature, the radioelement is distributed according to Doerner-Hoskins' logarithmic law, viz. $\ln \frac{c}{c_0} = \lambda \ln \frac{a}{a_0}$ (3).

Hence some investigators have concluded (4) that the logarithmic law expresses the true equilibrium, while Berthelot-Nernst's law is a non-equilibrial one, only formally identical with the thermodynamic law. This conclusion cannot be true, however, since Berthelot-Nernst's distribution is attained through a lengthy recrystallization from two opposite sides, which is a normal physico-chemical proof of equilibrium.

On the other hand, Doerner-Hoskins' distribution, which is attained through slow evaporation, cannot be an equilibrial one, because if the crystals are rich in the microcomponent ($D>1$) the quantity of the microcomponent in the solution diminishes more rapidly than the volume of the solution. The subsequent layers of the crystals grow from a solution containing less and less of the microcomponent and the concentration of the latter decreases from the centre of the crystal to the periphery. In the opposite case ($D<1$) the concentration of the microcomponent increases from the centre to the periphery.

Integrating Berthelot-Nernst's law for each layer of the crystal, we get the logarithmic law for the whole crystal:

$$\frac{dc}{da} = \lambda \frac{c}{a};$$

$$\int_{c_0}^{c} \frac{dc}{c} = \lambda \int_{a_0}^{a} \frac{da}{a}; \qquad \ln \frac{c}{c_0} = \lambda \ln \frac{a}{a_0}.$$

This distribution is not an equilibrial one, however, since the concentration of the microcomponent in different layers of the crystal is different. Only the external layer is in equilibrium with the solution. The absence of diffusion in the crystals at ordinary temperatures prevents the concentration of the microcomponent in the crystals from equalizing and renders equilibrium between the whole crystal and the solution unattainable. As a result of this we get a «frozen» false equilibrium, and not a thermodynamical one.

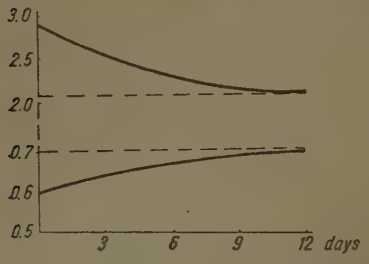

Fig. 1.

The absence of diffusion in the crystal can be substituted by prolonged recrystallization; in this case the concentration of the microcomponent inside the crystals should equalize and the «average» value of D, calculated for the initial «frozen» crystals, should fall when $D>1$ and rise when $D<1$, until the true distribution coefficient is attained.

I conducted such experiments for the systems $BaCl_2$—Ra ($D=2.1$) and $Ba(NO_3)_2$—Ra($D=0.68$).

The results are given in the table and in the figure. From these results it is obvious that the initial non-equilibrial value of D gradually approaches its true value.

Recrystallization of «Frozen» Crystals of $BaCl_2.2H_2O$—Ra in a Saturated Solution of $BaCl_2$—Ra at 100°

Volume of liquid phase, cm³	Volume of solid phase, cm³	Duration of recrystallization, days	D
100	7.45	0	2.72
100	7.45	4	2.44
100	7.45	8	2.09
100	7.45	12	2.15

Recrystallization of «Frozen» Crystals of $Ba(NO_3)_2$—Ra in a Saturated Solution of $Ba(NO_3)_2$—Ra at 100°

Volume of liquid phase, cm³	Volume of solid phase, cm³	Duration of recrystallization, days	D
100	8.75	0	0.59
100	8.75	3	0.63
100	8.75	6	0.67
100	8.75	12	0.68

It is quite obvious that Berthelot-Nernst's distribution between solutions and crystals is an equilibrial one, while the logarithmic distribution is a non-equilibrial, «frozen» one.

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Received
26. V. 1938.

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1938. Volume XX, № 4

CHEMISTRY

STABILITY OF CO-ORDINATED ETHYLENE HYDROCARBONS

By ANNE GELMAN

(Communicated by N. S. Kurnakov, Member of the Academy, 11. VI. 1938)

A study of the ethylene, propylene, butylene and styrene compounds of platinum (1) gave us the idea of at least qualitively comparing the stability of co-ordinated ethylene hydrocarbons in the hope of throwing light on the mechanism whereby the co-ordinate linkage between ethylene hydrocarbons and platinum is formed.

In his latest work J. Stuart Anderson (2) described one of the most interesting «methods» of obtaining complex compounds with unsaturated hydrocarbons, i. e. by replacing one hydrocarbon by another.

We decided to use this method in order to make a successive comparison of the stability of co-ordinated ethylene hydrocarbons, in view of the fact that hydrocarbon will possess a greater co-ordinating power, which is capable of uniting directly with the platinum ion, displacing another hydrocarbon, weaker in this respect.

An acid solution of Zeise's propylene salt was treated for four days with ethylene under a little pressure (from a gasometer). As a result the following reaction took place:

$$NH_4[C_3H_6Cl_3Pt]+C_2H_4 = NH_4[PtC_2H_4Cl_3] + C_3H_6 \uparrow .$$

When pyridine was added to this solution there appeared a precipitation of platinum ethylenepyridinedichloride:

$$NH_4[PtC_2H_4Cl_3] + Py = \downarrow [PtPyC_2H_4Cl_2] + NH_4Cl.$$

Zeise's salt was again produced according to the reaction

$$[PtPyC_2H_4Cl_2] + HCl \longrightarrow PyH[PtC_2H_4Cl_3]$$

and treated under the same conditions with propylene for four days. Then it was precipitated with pyridine:

$$\text{I. } PyH[PtC_2H_4Cl_3] + C_3H_6 \longrightarrow PyH[PtC_2H_4Cl_3] + C_3H_6 \uparrow$$
$$\text{II. } PyH[PtC_2H_4Cl_3] + Py \rightarrow \downarrow [PtPyC_2H_4Cl_2] + PyHCl.$$

An analysis of this precipitate showed that there was absolutely no replacement of ethylene by propylene.

This substance was dissolved in hydrochloric acid, and with Reise's salt of the former base gave a typical zeisite precipitate:

$$[Pt(NH_3)_4][PtC_2H_4Cl_3]_2.$$

These reactions lead us to the conclusion that while ethylene displaces propylene from its compounds, propylene is unable to displace ethylene. The following experiments confirmed this conclusion.

Potassium chloroplatinite was treated for five days with a mixture of equal volumes of propylene and ethylene. The solution (which turned yellow) was filtered off from the small quantity of platinum and was precipitated with ammonia. The yellowish grey precipitate was washed with water, dissolved in hydrochloric acid, filtered, and again precipitated with ammonia. Finally an absolutely pure substance of lemon-yellow colour was obtained.

From the analysis it may be inferred that the reactions took place according to the equations:

$$\text{I. } K_2PtCl_4 + C_2H_4 + C_3H_6 = K[PtC_2H_4Cl_3] + C_3H_6 + KCl.$$
$$\text{II. } K[PtC_2H_4Cl_3] + NH_3 = [PtC_2H_4NH_3Cl_2] + KCl.$$

0.775 g $[PtC_2H_4NH_3Cl_2]$ was obtained from 2 g of K_2PtCl_4. The substance yield is 52% of what it should be according to theory. This is an interesting case of ethylene catalysis with propylene as the catalyst. If the chloroplatinite is treated the same length of time with ethylene alone, the yield generally does not exceed 15%, while when it is treated with a mixture of ethylene and propylene the yield rises to 52%.

It may be supposed that in the beginning propylene, being the more soluble gas (it is about twice as soluble as ethylene), reacts more quickly with the chloroplatinite. But when the solution becomes sufficiently saturated in ethylene, the latter displaces propylene from the complex. Now the displacement goes on more quickly than the addition of ethylene to the platinum ion. All this taken together accelerates the formation of Zeise's ethylene salt; propylene is here a somewhat peculiar catalyzer, clearing a place in the complex for ethylene.

If Cossa's salt is treated with the same gas mixture, a corresponding ethylene cis-compound is the result.

$$NH_4[NH_3Cl_3Pt] + C_2H_4 + C_3H_6 = [PtC_2H_4ClNH_3Cl] + C_3H_6 + NH_4Cl.$$

From this we conclude that ethylene is more stable than propylene with regard to co-ordination.

Experiments were conducted with a mixture of ethylene and butylene under similar conditions:

$$\text{I. } K_2PtCl_4 + C_2H_4 + C_4H_8 = K[PtC_2H_4Cl_3] + C_4H_8 + KCl.$$
$$\text{II. } K[PtC_2H_4Cl_3] + NH_3 = [PtC_2H_4NH_3Cl_2] + KCl.$$

When Cossa's salt and chloroplatinite were treated with a mixture of ethylene and butylene, only ethylene compounds of the trans- and cis-forms respectively were obtained. The ability of ethylene to co-ordinate is therefore above that of the butylene.

It was interesting to compare the co-ordination stability of propylene and butylene from this point of view. We obtained propylene salts by treating Zeise's butylene salt with propylene:

$$\text{I. } K[PtC_4H_8Cl_3] + C_3H_6 = K[PtC_3H_6Cl_3] + C_4H_8 \uparrow.$$
$$\text{II. } K[PtC_3H_6Cl_3] + Py = [PtC_3H_6PyCl_2] + KCl.$$

This gave us grounds to assume that propylene exceeds butylene in co-ordinating ability. When, however, we treated Zeise's propylene salt with butylene and tried to precipitate it with pyridine, we got only a sticky, oily mass, which is characteristic of butylene compounds:

$$\text{I. } K[PtC_3H_6Cl_3] + C_4H_8 = K[PtC_4H_8Cl_3] + C_3H_6 \uparrow.$$
$$\text{II. } K[PtC_4H_8Cl_3] + Py = [PtC_4H_8PyCl_2] + KCl.$$

Thus, the co-ordination stability of propylene and butylene is almost equal and their mutual replaceability is merely an evidence of the following equilibrium being established in the solution:

$$Me_2PtCl_4 + C_nH_{2n} \rightleftarrows Me[PtC_nH_{2n}Cl_3] + MeCl.$$

When the co-ordinating abilities are equal, the gas that takes part in the direct reaction is mainly the one, which has the greater mass action in solution. If the propylene salt be treated with butylene, the concentration of butylene in the solution will be greater, and it will gradually displace the propylene. If there is more propylene in the solution, however, then butylene is displaced. When we added styrene to Zeise's propylene salt in the cold, gas bubbles rapidly generated. It may be supposed that propylene is evolved and that the reaction goes on according to the equation:

$$NH_4[PtC_3H_6Cl_3] + C_8H_8 = NH_4[PtC_8H_8Cl_3] + C_3H_6 \uparrow.$$

When the gas had stopped generating and the solution was treated with ammonia, it at once precipitated orange-coloured fine crystals:

$$NH_4[PtC_8H_8Cl_3] + NH_3 = [PtC_8H_8NH_3Cl_2] \downarrow + NH_4Cl.$$

The precipitate was washed several times with water, sucked off and dried over sulphuric acid.

In composition, appearance and properties this substance was absolutely identical with the $[PtC_8H_8NH_3Cl_2]$ obtained formerly (1).

Styrene displaced propylene quickly and entirely at room temperature; therefore we may conclude that it is co-ordinately stronger than propylene.

It was curious to compare the strength of styrene and ethylene under the same conditions.

Anderson (2) discovered that if an alcoholic solution of Zeise's salt is warmed in vacuum with styrene, the latter replaces ethylene quantitatively. The author draws the conclusion that at atmospheric pressure ethylene stays in the complex more stably than other olefines, while under decreased pressure it entirely drops out of the system. Consequently he places ethylene ahead of styrene in co-ordination stability.

We prepared an acid solution of Zeise's salt as formerly, and added a few drops of styrene. Gas bubbles slowly began to generate. After twenty minutes we added ammonia. A yellow precipitate immediately appeared. After standing half an hour together with the filtrate, the precipitate turned orange:

$$I.\ Me[PtC_2H_4Cl_3] + C_8H_8 = Me[PtC_8H_8Cl_3] + C_2H_4 \uparrow.$$
$$II.\ Me[PtC_8H_8Cl_3] + NH_3 = [PtC_8H_8NH_3Cl_2] + MeCl.$$

After being filtered, the precipitate was well washed with water, dried over H_2SO_4 and analyzed.

The 1.08% discrepancy between the theoretical content of platinum and that observed shows that only a very small part of ethylene was not displaced by styrene during this time. It is quite evident that at room temperature and atmospheric pressure styrene displaces ethylene from the complex, atlhough with greater difficulty than in the case of the propylene salt. On this ground styrene should be considered superior to ethylene in respect of co-ordinate linkage. To decide, however, finally whether C_2H_4 or C_8H_8 should be placed first, more experiments under increased pressure are needed, which will place C_8H_8 and C_2H_4 under the same conditions as to their concentra-

tion in the solution. From the experimental material here discussed the ethylene hydrocarbons should be arranged in the following sequence with regard to their co-ordination stability:

$$\begin{matrix} H \\ C_6H_5 \end{matrix}\!\!>\!C = CH_2 > C_2H_4 > C_3H_6 \backsim C_4H_8.$$

It is clear that this sequence, based only on experimental data, is in need of theoretical grounding and generalization. It would, however, be somewhat premature to do this yet; all the more so, since recently, while investigating platinum carbonyl compounds, we observed a complete replacement of ethylene by carbon monoxide according to the equation:

$$\text{I. } Me[PtC_2H_4Cl_3] + CO = Me[PtCOCl_3] + C_2H_4\uparrow.$$
$$\text{II. } Me[PtCOCl_3] + Py = [PtCOPyCl_2] + MeCl.$$

Carbon monoxide is not replaced by styrene.

We shall write in more detail on carbonyl compounds in the next article. From a practical viewpoint this sequence is certainly interesting even now. It can be used to separate a mixture of ethylene hydrocarbons, to detect ethylene in a gaseous mixture with the help of a solution of Kossa'salt, etc.

If cheaper absorbents than platinum salts were to be found, this method of separating ethylene hydrocarbons could be successfully applied in our industry.

Conclusions

1. The stability of co-ordinated ethylene hydrocarbons decreases under normal conditions according to the sequence:

$$C_2H_2 > C_2H_4 > C_3H_6 \backsim C_4H_8.$$

2. The co-ordination stability of carbon monoxide is greater than that of the ethylene hydrocarbons.

3. Cossa's salt may be used as a reagent for ethylene even when the latter is in a gaseous mixture (C_2H_4, C_3H_6, C_4H_8, CO).

Institute of General and Inorganic Chemistry.
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Moscow.

Received
3. VI. 1938.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

GEOCHEMISTRY

ON CAESIUM AND LITHIUM IN THE ILMEN RANGE IN THE URALS

By A. F. SOSEDKO

(Communicated by A. E. Fersman, Member of the Academy, 8. VI. 1938)

It has been repeatedly reported (1) that the highest concentration of caesium in nature is confined to low temperature pegmatites, characterized by the presence of lepidolite, pink tourmaline, etc. Just here all the principal caesium minerals (pollucite with 30% Cs_2O, vorobyevite with 3% Cs_2O, lepidolite with 0.8% Cs_2O) use to be isolated. For this reason the author made the suggestion to search for caesium raw material first of all in the lepidolite deposits of the USSR.

The pegmatites of the Ilmen mountains in the Urals have not been discussed in those papers. This is accounted for, first, by the fact that the caesium content in the minerals of these mountains was reported as being very low (amazonite 0.03—0.09% Cs_2O), and, secondly, that the low temperature phases of the pegmatite process have not been described at all.

The somewhat unusual isolation of caesium during the earlier phases of the pegmatite process in the Ilmen mountains could, however, not remain unnoticed.

In the Ilmen mountains caesium has been encountered in amazonite at first qualitatively (V. I. Vernadsky); later on quantitative analyses have been made by different investigators. The caesium content is low, varying from 0 to 0.09% (analyses made by Filippov and Tolmachev) and in one instance only has shown 0.6196% $Cs_2Al_2Si_6O_{16}$ or about 0.18 Cs_2O (Milchevskaya-Rutkovskaya). In spite of the fact that rubidium has been found in amazonite too, and that the quantity of amazonite is in general very considerable, the low caesium content and the difficulty of its recovery prevented amazonite from becoming important from the economical viewpoint as a caesium raw material.

Caesium containing amazonite has been encountered in the Ilmen range in pegmatite veins of the granite type. These pegmatites are observed almost exclusively in the eastern spurs, named Kossaya Gora. On the whole 50 amazonite veins have been recorded. The pegmatite veins in which amazonite has been discovered consist of several zones, characterized by a definite mineralogical composition. These zones are as follows:

1) The aplite zone, the earliest as to the time of formation, corresponds to A. E. Fersman's geophase *B*; it is developed in narrow bands along the marginal contacts.

2) The graphic granite zone, corresponding to the geophase *C*, highly developed in certain veins.

3) The pegmatoid zone with microcline and quartz—geophase *D*.

4) The amazonite zone, also geophase *D*.

5) The albite zone, corresponding to the geophases *E* and partly *F*, has always been encountered together with the amazonite zone. The appearance of this geophase is characterized by a replacement of previously formed minerals, especially amazonite, beginning along the cleavage cracks and ending in a complete replacement leaving out only small areas.

6) The cryolite zone. This zone is the latest corresponding to A. E. Fersman's geophases *G—H*. It was encountered almost 90 years ago in one vein only (vein № 69), and was completely worked out at the time.

Approaching the analysis of the Ilmen pegmatites from the viewpoint of caesium ditribution, it should be noted first of all that caesium is present in the amazonite of earlier zones than those observed in other pegmatites. An isomorphous replacement of K and Rb evidently takes place in this case. This circumstance permits of the following statements.

1. The penetration of caesium into the lattice of minerals during the earlier stages of the pegmatite process might be explained by the somewhat higher caesium concentration in the pegmatite melt.

2. Probably, not the whole amount of caesium has been precipitated from the melt during the process of amazonite formation and a portion might have been retained in the melt after the crystallization of the amazonite phase.

3. The replacement of amazonite by later minerals of the albite phase contributed to the migration of caesium from the amazonite to the melt, thus enriching the latter in caesium.

4. From the analogy with other pegmatites the presence of caesium-bearing minerals should be expected in the low temperature phases, in the present case in the cryolite zone.

5. Alongside of this, caesium might be expected to be found in those minerals of the albite zone which contain potassium, since potassium and caesium may be replaced isomorphously.

In the course of field work all the amazonite veins have been examined as far as possible, and material on the mineralogy of the albite zone of these pegmatites has been collected. In the field a black lamellar mineral at once attracted our attention, which has been encountered in all the pegmatites examined. It has been precipitated after the albite, filling small cavities in the latter. This mineral has been previously recorded by Melnikov ([2]), E. Kopteva-Dvornikova ([3]) under the name of black mica or biotite. Its occurrence with albite is quite unusual, and called forth a natural remark from A. E. Fersman that he has never observed biotite together with albite.

By a spectral analysis made in the Laboratory of the State Institute of Rare Metals in Moscow this biotite was shown to contain a high quantity of caesium and rubidium and a quantity of rare alkalies higher than that in amazonite.

Thus, the theoretic assumptions which controlled the search have been fully confirmed, and the mineral recorded by Kopteva-Dvornikova under the name of biotite is evidently either a new mineral or the mineral described by Hess under the name of caesiobiotite. The mineralogical study of this mineral is the subject of a special work of the author.

A parallel search for the cryolite zone has been conducted. The area around the «Cryolite» mine has been thoroughly searched. So far the search is not completed, since cryolite is difficult to determine. It is necessary to develop simple and quick methods for the field determinations of cryolite. Probably, the most effective method will be the exposure of the specimens to the action of ultra-violet rays.

In the course of search attention has been paid to pink mica. The latter has been encountered in a number of mines in paragenesis with feldspar, corundum, zirconium. Spectroscopic investigations carried out in the Laboratory of the Institute of Rare Metals showed the presence of lithium in this mica in quantities approaching lithium micas. Pink micas from mines № 68 and 98 have been analyzed.

Aside from these mines, pink mica has been discovered in blocks, between the cryolite mines № 69 and 68; in a number of corundum mines near lake Ishkul. Besides lithium has been discovered spectroscopically in «Barbot's eye» mica (mine № 60), in light-blue amazonite from the Prutov mine, in muscovite from mines № 50, 56 and in deep-green aquamarine (mine № 38).

These data bear testimony to a much greater distribution of lithium in the Ilmen pegmatites than it has been known till now. Up to the present time lithium has been reported in cryolithionite ([4]), encountered together with cryolite, and in small quantities in green mica from the corundum pegmatite in the Maly Ilmen Range ([5]).

The observations made confirm the conjectures as to the widespread development of lithium in the Ilmen pegmatites.

At the same time the discovery of lithiun shows that not all by far has been done in the domain of rare alkalies discovery and that a number of new discoveries is to be expected.

All the above data lead to the conclusion that the problem of caesium at the Ilmen Range is to be solved rather through a search for minerals enriched in caesium during later phases of pegmatite formation, than by way of amazonite. The findings of lithium and of caesium-bearing «biotite» and a number of geochemical observations bear evidence in favour of the above conclusion.

It thus becomes quite obvious what is to be done in this field for the nearest future. The first task is a study of «biotite» for its caesium content; later on search for pegmatites enriched in caesium-bearing «biotite» and, finally, a thorough search for cryolite zones in pegmatites is to be conducted. To discover the latter, the method of topo-mineralogical survey must be used which has been already suggested by V. I. Vernadsky, and a rapid method of cryolite determination by exposure to the action of ultra-violet rays is to be developed.

The Uzbekistan State University.
Samarkand.

Received
8. VI. 1938.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

BIOLOGY

NEW DATA ON REVITALIZATION OF ORGANISMS FROM PERPETUALLY FROZEN GROUNDS

By P. KAPTEREV

(Communicated by V. I. Vernadsky, Member of the Academy, 25. V. 1938)

Since the results of my first experiments on revitalization of organisms from perpetually frozen ground had been reported, I succeeded in performing another series of experiments and in accurately determining the material obtained. Samples were taken at the station Skovorodino of the Amur railroad. The station is located in the valley of the Bolshoy Never river (left-hand tributory of the Amur) and, according to the geologist A. V. Lvov, it is of fault origin. It is 2—3 km wide. No terraces have been observed. The base of the valley is composed of metamorphized shales and partly sandstones which seem to be of Jurassic age, though exact determination was prevented by the absence of fossils. The upper sheet of the valley is of alluvial origin and at places attains a thickness of 20—25 m. The thickness of the frozen ground is about 60 m, the depth of summer-melting is about 2.5 m. In different horizons have been found of the alluvial bed slime inclusions with vegetable remains and even turf. In the test-pit, for instance, made in October 1937, 3.9 m deep, five layers of slime with vegetable remains have been discovered, even with well preserved fir cones. In the course of boring on the territory of the scientific-research station in 1931, several buried humus horizons have been discovered, namely at a depth of 3—3.5 m clay of ashen colour with humus stains and half decomposed vegetable remains, a similar thin band at a depth of 405 cm; 9.2—9.7 m deep a layer with well preserved remains of herbaceous plants has been found. A very condensed humus horison is lying at a depth of 10.2—10.7 m. The latter was hardly accessible by bit drilling.

Stumps of old larches have been preserved in the area of the Frozen ground station as well as below and beyond the places where samples were taken out. According to our determinations made in 1937 some of the stumps are as old as 400 years. The samples of 1934—36 were taken at a depth of 2.75—6 m over an area somewhat larger than 200 m. After thawing the samples of these frozen slime were poured over with distilled water. From these samples a number of organisms, chiefly algae and partially mosses, were restored to life. They could, however, not be determined on account of the absence of fruiting. The algae obtained from 42 samples of slimy ground have been determined by professor K. I. Meyer. A list of these is given below.

Chlorophyta

1. *Chlamydomonas* sp.
2. *Schizochlamys gelatinosa* Abr.
3. *Chlorococcum infusionum* (Schrank) Menegh.
4. *Chlorococcum humicola* (Naeg.) Raben.
5. *Stochococcus bacillaris* Naeg.
6. *Chlorella vulgaris* Beyer.
7. *Ulothrix subtilissima* Raben.
8. *Ulothrix tenerrima* Kütz.
9. *Microspora willeana* Lagerst.
10. *Microspora tumidula* Hazen.
11. *Stigeoclonium* sp.
12. *Oedogonium* sp. sp.
13. *Cladophora fracta* Kütz.
14. *Mugeotia* sp.
15. *Closterium moniliferum* (Bory) Ehr.
16. *Cosmarium Meneghinii* Bred.

Xanthophyta (Heterocontae)

1. *Tribonema bombycinum* Berbes & Solier.
2. *Tribonema minus* (Wille) Hazen.
3. *Tribonema affine* G. D. West.
4. *Tribonema tenerrimum* (Gay) Heering.
5. *Botrydiopsis arrhiza* Borzi.

Bacillariophyta

1. *Synedra Ulna* (Nitzsch.) Ehr. var. *oxyrhynchus* (Kütz) v. H.
2. *Navicula crystocephala* Kütz.
3. *Navicula tuscula* (Ehr.) Grun.
4. *Neidium bisulcatum* (Lagerst) Cl.
5. *Pinnularia microstauron* (Ehr.) Cl. f. *diminutum* Grun.
6. *Pinnularia gibba* (Ehr.) v. *linearis* Hust.
7. *Stauroneis anceps* Ehr.
8. *Comphonema constrictum* Ehr.
9. *Nitzschia palea* (Kütz) W. Sm.
10. *Surirella angustata* Kütz.

Cyanophyta

1. *Chroococcus minutus* (Kütz) Naeg.
2. *Oscillatoria tenuis* (Menegh.) Gom.
3. *Chroococcus turgidus* (Kütz) Naeg.
4. *Phormidium tenue* (Menegh.) Gom.
5. *Phormidium purpurascens* (Kütz.) Gom.
6. *Phormidium valderianum* (Delp.) Gom.
7. *Phormidium ambiguum* Gom,
8. *Phormidium Retzii* (Ag-) Gom.
9. *Phormidium laminosum* Gom.
10. *Phormidium viride* (Vauch) Lemm.
11. *Phormidium molle* Gom.
12. *Lungbya aerugineo-coerulea* (Kütz) Gom.
13. *Nostoc sphaericum* Vauch.
14. *Nostoc paludosum* Kütz.
15. *Nostoc muscorum* Ag.
16. *Cylindrospermum majus* Kütz.
17. *Anabaena variabilis* Kütz.
18. *Anabaena* sp.

This list is far from comprising all the organisms, which were by far more numerous. Among them were infusoria and *Chydorus sphaericus* of the *Crustacea*. Unfortunately, number of forms disappeared before they could be determined. Some of them existed not more than 24 hours.

As it may be seen the flora of shallow layers of the frozen ground does not present any peculiarities. True enough, certain forms, e. g. *Chlamydomonas*, *Chaetophora*, *Stigeoclonium*, *Anabaena* and some others, have been difficult to determine precisely, but this might possibly depend upon some peculiar conditions of their development. It should, however, not be for-

gotten that forms morphologically identical to those known by us, may be quite different from the biological viewpoint. We thus know nothing about the forms of water in connection either with recent, or ancient organisms of those regions. Since samples were taken from a shallow depth, there was no reason to expect the finding of any «fossil» forms.

In 1936 samples were taken at a depth of 7 m from a test-pit made in the course of construction of a school-house in Skovorodino. These dried samples in the form of small monoliths were brought to Moscow, and here they have been subjected to microbiological investigations by D-r N. A. Krassilnikov (Institute of Microbiology of the Academy of Sciences of the USSR). Repeated inoculations were done in different nutritive mediums and gave the following results: no nitrogenfixing, nitrifying or cellulose bacteria have been detected. Common forms of mould have been isolated from fungi, which evidently should be referred to recent forms.

Spore-bearing bacteria are prevailing. Judging by their physiological properties, they sharply differ from those found in Central Russian grounds. They possess no ammonifying ability, nor do they form ammonia in protein media; no proteolitic capacity is displayed either; they do not dilute gelatines nor peptize milk, but provoque pronounced coagulation of the latter.

All the strains have been feebly developing in meat—pepton—agar and other mediums, most of them having perished at the 3—5 inoculation.

The following fungi have been detected: *Mucor racemosus*, *Mucor stolonifer*, *Trichoderma*, *Cephalosporium*, which are of no particular interest. The following bacteria have been determined:

Bacillus coagulans Hammer (?)
Bacillus sphaericus Neide (?)
Bacillus mirulans Boucquest (?)
Bacillus sublustris Schieblich (?)
Bacillus nondiastaticus Bergey (?)
Mycobacterium globiforme Conn.
Mycobacterium album Krassiln.
Bacterium hyalinum Bergey.

Five species of bacteria are put with an interrogation point, since the exactness of their determination lacks security. They physiologically differ from corresponding forms studied in the temperate zone. Besides four species of bacteria altogether refused determination; perhaps these are new species. We must, however, make no hasty conclusions, since the present-time surface microflora of those regions has not been studied as yet. It might be found that similar bacteria are living even now on the surface of the ground at Skovorodino. Surface ground samples were brought by the author in November 1937 and are studied at present in the Institute of Microbiology of the Academy of Sciences of the USSR.

Probably, we are rather to do with oxbows or pools near the river, which has at present diverted from the platform of the station at a distance of almost 2 km. If this be true, the age of the alluvial beds out of which the samples were taken is considerably greater than it was previously supposed (1—3 thousands years). The East-Siberian and Far East-expedition of 1938 gave new valuable material, the treatment of which will soon be accomplished.

Committee for the Study of Perpetually Frozen Gronnds.
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Received
26. V. 1938.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

GENETICS

STUDIES IN GENETICS OF DEVELOPMENT. II. INFLUENCE OF LETHAL GENES ON THE DEVELOPMENT OF CHARACTERS AS STUDIED BY TRANSPLANTATION

By N. N. MEDVEDEV

(Communicated by N. I. Vavilov, Member of the Academy, 25. V. 1938)

Problem

The necessity and importance of working out a theory of phenogenetics, or developmental genetics, is so evident that it hardly needs to be discussed here. Further disregard for this field of investigation would lead to a still greater increase of the gap between two intimately connected sides of the whole problem of heredity, namely, the genetics of development and the chromosome theory of inheritance dealing with the mechanism of transmission of genes among the offspring of successive generations of individuals.

The reason why work in the field of developmental genetics was lagging behind should be referred to the fact that genetics, on the one hand, and embryology, dealing with the problems of development of the individual, on the other, have gone their respective ways in considerable isolation from each other. The two fields differ widely both in subject and methods used, although these differences are to some extent justified by the logical development of these branches of biology.

Another no less important obstacle to progress in developmental genetics has been a lack of methods of investigation efficient enough to permit of a direct approach to the problems whose solution has long since become an evident necessity.

Some advance in phenogenetical research methods came, however, in 1935 with Beadle and Efrussi's application of transplantation technique to *Drosophila* material. This fly has been so thoroughly studied genetically and has such large numbers of mutations of various types and nature, that it is a highly suitable object for the investigation of many problems in developmental genetics.

Using the transplantation method, we have attempted to work out some general problems bearing on the mechanism through which genes are controlling the development of characters. As material for the present investigation we took true point lethals, as being a priori the most heterogeneous group of mutations both genetically and, in all probability, in respect to their mode of expression during ontogenesis.

The present communication is devoted to a description of the experiments of transplanting to normal larvae the imaginal discs and gonads of larvae carrying lethal genes and usually dying at the pupal stage of development.

Material and Methods

The material on which the present investigation is based consisted of 30 stocks of *Drosophila melanogaster* bearing lethal genes in the *X*-chromosomes. These stocks were given us by V. V. Sakharov (of the Institute of Experimental Biology, Moscow). All these mutations occurred spontaneously and were found by Sakharov in the control material of his experiments on the influence of various chemicals on mutation frequency.

As Sakharov's genetical and particularly cytological studies of salivary gland chromosomes in these stocks showed, all these spontaneous mutations are accompanied by no visible changes in the structure of the *X*-chromosome. We therefore agree with Sakharov in regarding them as true point mutations.

As stated above, all the lethal mutations in question were localized in the *X*-chromosome. To exclude possible loss of lethal genes due to crossing-over, such stocks are usually maintained by introducing a suppressor of crossing-over (inversion) into the homologous *X*-chromosome of the female. In our case such a balancing chromosome was $ysc^8w^al z^s del\ 49$ with double inversions, completely preventing crossing-over in the *X*-chromosome.

First of all we determined for each stock the particular stage at which the lethal gene exerted its influence and caused the death of the lethal zygotes. With this purpose in view, we introduced into the balancing *X*-chromosome the gene for yellow body colour whose influence can be recognized since very early stages of development. Owing to this peculiarity the two kinds of male larvae, i. e. the grey (lethal) and yellow (viable), arising from heterozygote females, having the constitution $\left(\frac{l}{ysc^8w^al z^s del 49}\right)$, can be easily distinguished since very early larval stages.

Our investigations showed that in 15 of the 30 stocks the lethal zygotes die at the pupal stage, while in the other 15 stocks death occurs either before the hatching of the larvae from the eggs or during the larval stage (i. e. before pupation).

After determining for each stock the particular stage at which the lethal zygotes die and selecting a number of races in which the lethal gene caused death in the pupal stage, we began a series of experiments, transplanting different imaginal discs and gonads from such lethal larvae to normal ones. As host larvae we used the *wild type* (*Berlin*) of *white* and *yellow white*.

Transplantations of imaginal discs and other organs was always carried out at the stage of adult larvae, that is just before the larvae begin their pupation, when they stop feeding and crowling about on the walls of the culture bottle. Where necessary, drawings were made which, however, could not be given here for lack of space.

The preliminary work which consisted in balancing the stocks and preparing the material for operations was carried out by I. K. Schmidt.

Results and Conclusions

The results obtained in the experiments under consideration are briefly as follows.

1. Imaginal discs from lethal larvae when transplanted to normal larvae develop quite normally in all cases thus far examined. Implanted eyes

developing from lethal imaginal discs differ in no way from controls, that is from implants developing under the same conditions from normal imaginal discs.

2. The same is true for all the other transplanted rudiments examined by us, such as wings, legs, etc.

3. The most interesting result, however, was obtained in similar experiments with transplantation of testes of lethal larvae, where their development and differentiation was more or less normal. Furthermore, when implanted testes were attached to the ducts of the host they replaced the testes of the latter, being developed in a perfectly normal way.

4. The implanted testes frequently became attached to the host testes; the result was a peculiar mosaic testis of the type recently described by Stern and Hadorn in similar experiments with transplantation of testes in *Drosophila melanogaster* and by Dobzhansky in his work on gynandromorphs of *Drosophila simulans*.

5. When the implanted testes lie free in the body cavity, they retain, as a rule, the juvenile form.

6. During the work a number of lethal mutants were found in which various internal organs had undergone marked changes. Thus, in one such case, the lethal alteration resulted in the occurrence of malignant tumors of the intestine. In another case, the gonads were reduced to very small vesicles which, moreover, seemed to contain no spermatogonia.

7. The most obvious conclusion to which the above data lead is that the mode of action of lethal genes is in general the same as that of genes for visible characters. Their developmental influence is localized in the sense that only certain organs or rudiments of organs in the embryo are affected by the particular lethal gene, while the great majority of body cells are just as viable as those containing no lethal genes. Thus, the fatal effect of lethal genes is not of cellular nature, the death of the organism being evidently due to disturbances in the normal functioning of the organ affected.

These facts tend to delete the border line between visible and lethal mutations and show that there is no difference in principle between the two types of hereditary changes in either their mode of origin and further behaviour in subsequent generations, or in their mode of action during development.

Further study of such cases will enable us to determine more exactly the causes of death of lethal zygotes and in some cases to render such zygotes again viable by substituting the normal organs for the affected ones.

At present similar studies of point lethal mutations killing the zygote in the larval and egg stages, as well as of various other types of lethal mutations, involving visible changes in the structure of chromosomes, are in progress.

Institute of Genetics.
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Moscow.

Received
1. VI. 1938.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

ANATOMIE DES PLANTES

DE L'INFLUENCE DU SÉCHAGE PHYSIOLOGIQUE DU BOIS DE HÊTRE SUR LE RÉGIME DE SES MATIÈRES PLASTIQUES

Par A. YATSENKO-KHMELEVSKY et H. KONUCHEVSKA

(Présenté par A. A. Richter, de l'Académie, le 10. VI. 1938)

Dans un article publié récemment par un des auteurs (1) nous avons analysé les résultats du «séchage physiologique» du bois de hêtre. Trois modifications du séchage physiologique ont été examinées: 1) le séchage par le sciage du tronc de l'arbre restant sur pied, 2) le séchage par la conservation de la cime sur l'arbre coupé et 3) le séchage combiné par le sciage préalable du tronc et le séchage subséquent par la cime conservée.

Il a été établi que le sciage de l'arbre sur pied, trois mois après le sciage, n'exerce presqu'aucune influence sur l'humidité du bois. Au contraire, la conservation de la cime sur l'arbre coupé provoque un desséchement considérable du bois, en abaissant son humidité de 70% jusqu'à 40—45%.

Il était intéressant d'observer les modifications amenées par le séchage physiologique dans l'état des matières plastiques du bois. Dans quelques articles publiés antérieurement (2,3,4) nous avons examiné le régime des matières plastiques du bois après la chute de l'arbre. Dans ces travaux nous avons constaté que la chute de l'arbre provoque la transformation de l'amidon contenu dans les éléments parenchymateux du bois en une matière d'une nature chimique indéterminée, dénommée par nous «matière traumatique du bois». Ce processus est accompagné par l'augmentation du sucre dans le bois, et dans quelques cas par l'apparition de rares thylles. Vu que le séchage physiologique du bois par son caractère diffère sensiblement du séchage ordinaire du bois, on s'attendait à des différences dans les processus se déroulant dans les cellules vivantes du bois.

Les échantillons pour l'examen microscopique étaient pris immédiatement après la fin du séchage simultanément avec la prise des échantillons pour la détermination de l'humidité du bois. Les échantillons étaient pris en forme de ronds, hauts de 3—5 cm, pris dans toute la section du tronc. Ces ronds étaient pris à une distance de 0.5, 1.0, 1.5, 4.5 et 7 m du gros bout du tronc. De chaque rond on prenait à la hache deux échantillons, l'un à la périphérie du rond (dans l'aubier), l'autre au milieu du rayon (dans le bois parfait), et, de plus, dans les endroits avec une coloration du bois modifiée par l'attaque des champignons ou par d'autres causes. Les échantillons étaient immédiatement fixés à l'alcool. Simultanément avec l'analyse des troncs expérimentaux on prenait des échantillons du bois des arbres sur pied que l'on abattait à ce but et des grumes, laissées dans le taillis dans les

conditions ordinaires pour les mêmes laps de temps que les troncs expérimentaux. En tout nous avons examiné trente-neuf arbres et dix arbres ont été coupés pour servir de contrôle. Le séchage physiologique a été organisé deux fois. La première série des expériences fut commencée le 30 mai 1935; le séchage sur pied a duré jusqu'au 15 juillet (45 jours), le séchage par la cime jusqu'à la fin de juillet (60 jours) et le séchage combiné jusqu'au 5 octobre, c'est-a-dire 125 jours, dont 45 jours (jusqu'au 15 juillet) les arbres sont restés sur pied avec des traits de scie effectués à leurs bases, et du 15 juillet jusqu'à la fin de l'axpérience (80 jours) ont été laissés gisant avec leurs cimes. La seconde série fut ouverte le 20 juillet 1935. Le séchage sur pied a pris ici 90 jours, le séchage par la cime conservée aussi 90 jours, et le séchage combiné respectivement 64 jours (jusqu'au 24 septembre) et 30 jours (jusqu'au 24 octobre).

Les analyses effectuées selon les méthodes appliquées généralement dans la microchimie des plantes ont montré que le sciage du tronc des arbres sur pied n'influe presque pas sur le régime des matières plastiques comparées avec les arbres témoins. Le bois près des traits de scie a été infecté par les champignons. La zone affectée, comparativement étroite à la périphérie (dans l'aubier) s'agrandit dans la direction du centre. Il est probable que cela doit s'expliquer par le fait que le bois à la périphérie du tronc (l'aubier) est caractérisé par une humidité plus élevée et par une activité vitale plus intense, ce qui lui permet de former des thylles nombreuses (près de 50% de tous les vaisseaux dans la section transversale sont formés par les thylles) ce qui ralentit le desséchement du bois. On sait que l'umidité élevée du bois entrave la propagation des champignons ([5]). Dans les couches centrales, au contraire, où le bois est caracterisé par une humidité moindre et où les thylles moins nombreuses favorisent un desséchement plus rapide, les gyphes des champignons croissent plus vite. Une propagation analogue des champignons a été constatée par nous dans les grumes de hêtre laissées sur le taillis et atteintes par «l'échauffure» ([6]). Les échantillons pris dans le bois sain, en voisinage immédiat avec la zone infectée, ont montré une légère différence avec les échantillons pris dans les arbres témoins, en ce que les grains de l'amidon se coloraient moins vivement (une coloration lilas clair) sous l'action d'iode-iodure de potassium, ce qui peut être considéré comme le premier indice de la réaction traumatique ([2],[3]). Néanmoins, vu que les échantillons pris à une distance des traits de scie plus considérable (1 m et 4 m) furent tout-à-fait identiques aux échantillons de contrôle, on peut bien supposer que les différences notées ont été provoquées par le desséchement du bois par les traits de scie, desséchement qui devait avoir lieu, quoiqu'il ne fut presque pas observé dans nos déterminations de l'humidité.

Dans les troncs séchés par leurs cimes durant 2—3 mois, des différences notables ont été observées en fait de matières plastiques, en comparaison avec les arbres et les grumes témoins. Les arbres témoins, abattus vers la fin du séchage physiologique, ont montré dans leurs cellules parenchymateuses une quantité considérable d'amidon; les grumes de contrôle, débitées au moment du commencement du séchage et analysées vers la fin du séchage, montrent le processus de la transformation de l'amidon en matière traumatique du bois (dans les parties qui n'ont pas été affectées par l'échauffure). Dans le bois des arbres expérimentaux, au contraire, on a pu constater (dans les endroits qui n'ont pas été atteints par l'échauffure) une diminution saillante de l'amidon, une quantité minime de matières traumatiques et des thylles rares.

Le tableau général de l'état des matières plastiques du bois des troncs séchés par leurs cimes est assez compliqué, ce qui s'explique par la diversité des influences subies par le bois gisant 2—3 mois dans le taillis. Une partie

des troncs expérimentaux a été laissée avec l'écorce et a été attaquée par les bostryches, une autre partie a été privée de l'écorce et quelques troncs de ces derniers ont été badigeonnés a «l'ouralite» (moyen antiseptique). Une couche périphérique mince du bois des troncs avec l'écorce conservée a bruni fortement sous l'action de bostryches. Dans cette couche brunie on a pu observer une formation énergique de thylles, une transformation presque complète de l'amidon en matières traumatiques et l'augmentation du sucre. Les gyphes des champignons, notées dans ces couches, exercent probablement aussi une certaine influence sur les transformations constatées. Dans les troncs privés de l'écorce, le desséchement rapide de la surface mise à nu tue les cellules vivantes des couches périphériques et fixe aussi plus ou moins l'état des matières plastiques, grâce à quoi dans ces couches une partie de l'amidon, parfois assez considérable, ne se transforme pas en matière traumatique. Les thylles ici sont assez rares, et leur développement est arrêté à des stades premiers, c'est-à-dire ils ne ferment pas complètement la cavité des vaisseaux.

Les couches plus profondes du tronc qui n'ont pas été atteintes par l'échauffure, surtout dans les parties basales, diffèrent macroscopiquement du bois sain des grumes témoins par une coloration blanche. Néanmoins, cette différence, plus nette dans la partie basale, devient presque insensible dans les parties supérieures du tronc, car, comme il a été démontré par nos déterminations de l'humidité, le séchage physiologique exerce le plus d'influence sur les parties basales du tronc. L'analyse microscopique des parties, caracterisées par une coloration blanche, a montré une absence presque complète de l'amidon, une quantité insignifiante des matières traumatiques, surtout dans les coins des cellules, des thylles rares et le sucre dans des quantités presque les mêmes que dans le bois fraîchement coupé. Cet état des matières plastiques peut être observé surtout dans les parties basales du tronc et il devient bien moins saillant dans le voisinage de la cime. A une distance de 8—10 m du gros bout du tronc, l'état des matières plastiques du bois ne diffère presque guère de l'état des matières plastiques du bois sain des grumes de contrôle. Vu que l'humidité du bois des troncs séchés par leurs cimes s'élève dans la direction du gros bout jusqu'à la cime, il est naturel de supposer que la disparition de l'amidon observée est provoquée par le séchage physiologique du bois par la cime.

Dans nos expériences préalables sur le séchage physiologique par la cime, organisées en 1934, nous avons déjà observé cette disparition de l'amidon. Pour expliquer cette disparition nous avions supposé alors que l'amidon se transforme dans le bois en sucre et ce dernier est éloigné par le courant ascendant de la sève (7). Néanmoins, durant nos travaux de l'année 1935, le processus de la disparition du sucre n'a pas été observé avec les détails nécessaires et la supposition émise il y a quelques années ne peut être considérée que comme hypothèse privée de tout fondement expérimental.

Le prof. Junitzky, se basant sur nos données relatives à la disparition de l'amidon du bois séché par la cime, a affirmé dans plusieurs articles publiés dans la presse journalière que le bois séché de cette manière est plus résistant à l'attaque de champignons que le bois débité de la manière ordinaire. Néanmoins, nous avons pu constater que le bois séché par la cime est atteint par l'échauffure au même degré que le bois ordinaire.

Le tableau microscopique du bois échauffé, dans les parties basales des troncs expérimentaux, ne diffère presque pas de tableau de l'échauffure des grumes gisant dans le taillis, décrit par nous dans un article spécial (6). On doit expliquer ce fait par ce que l'échauffure dans ces endroits a eu lieu avant la disparition de l'amidon sous l'influence de l'action desséchante de la cime. A des distances plus grandes du gros bout, le

bois échauffé des troncs expérimentaux diffère de plus en plus du bois échauffé des grumes témoins en ce que les matières traumatiques et d'autres substances indéterminées qui caractérisent généralement l'échauffure (ainsi que les thylles) y sont moins nombreuses. Cela indique que l'échauffure s'est propagée dans le bois après la disparition de l'amidon. Dans la littérature technique et forestière on rencontre souvent des indications sur le rôle que l'amidon joue dans la diminution de la résistance du bois aux champignons destructeurs du bois ([8]). Il semble que cette assertion au moins en ce qui concerne l'attaque du bois de hêtre par les champignons qui lui sont propres (*Stereum hirsutum*, *Fomes fomentarius* etc.) n'est pas exacte. Lorsque le bois séché physiologiquement par la cime et par conséquent privé d'amidon fut transporté en hiver à la fabrique de meubles de Marelissi (République Soviétique de Géorgie), le sciage des grumes a montré que le bois a été presque complètement affecté par l'échauffure, au même degré que le bois des grumes ordinaires.

De ce qui a été exposé plus haut on peut faire les conclusions suivantes:

1. Le sciage des arbres sur pied ne provoque pas de changements considérables ni dans l'humidité du bois ni dans l'état de ses matières plastiques.

2. Le séchage par la cime, au contraire, abaisse sensiblement l'humidité du bois et provoque une diminution considérable de la quantité de l'amidon contenu dans ses cellules parenchymateuses.

3. La disparition d'une partie de l'amidon n'influe pas sur la résistance du bois aux champignons, car l'échauffure a lieu aussi bien dans le bois séché par la cime que dans le bois débité de la manière ordinaire.

4. Nos données indiquent que le sciage des arbres sur pied ne peut être recommandé pour le desséchement du bois; le séchage du bois par la conservation de la cime sur l'arbre abattu, au contraire, donne des résultats très saillants. Toutefois, vu qu'avec cette dernière opération s'impose la nécessité de laisser dans le taillis les troncs coupés pour un délai minime de 1—2 mois, il y a toujours un danger d'obtenir le bois plus ou moins sérieusement altéré par l'attaque des champignons. Il s'en suit que le séchage physiologique du bois de hêtre par la cime ne peut être recommandé que lorsque l'on doit à tout prix obtenir un bois desséché et, par conséquent, capable de flotter.

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Manuscrit reçu
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

ZOOLOGY

LIGHT AND THE PERIODICITY OF FORM-BUILDING PROCESSES IN BIRDS

By E. SVETOZAROV and G. STRAICH

(Communicated by S. A. Zernov, Member of the Academy, 1. V. 1938)

In the majority of ducks (*Anatidae*), alongside of clearly pronounced sexual characters, there exists a seasonal dimorphism. The colouring of the plumage in males changes twice a year, while in females the character of the plumag. remalns constant in spite of the moulting which takes place twice a yeare The wedding attire of the male is changed during the summer moult (June—Jule) to a modestly coloured plumage of the female type. The latter is cast off in autumn (September—October) and replaced by a brightly coloured male attire.

In our previous investigations([5]) we showed, first, that the summer (female) plumage of the drake is hormonally conditioned, and, secondly, that the process of its determination takes place only in the presence of a high activity of the gonads.

Under typical conditions the potency to develop a given type of plumage is realized in the course of the moult. The periodical activization of feather embryos is hormonally conditioned and is determined by a definite relation between the activities of the thyroid and sexual glands, both of them being controlled by the hypophysis. The activity of the thyroid gland, necessary to call forth the activization of feather embryos, is increased during the moulting period, while the activity of the gonads, which inhibit this process, is lowered ([6]). As far as both summer plumage and moult of the drake are depending upon the degree of sexual activity, we may expect under experimental conditions the following results:

1) The normal periodicity of the activity of gonads having been changed, this results in a corresponding shift of the moment of appearance in the feather-forming tissue of potency for the development of summer plumage, as well as in a shift of the moulting period.

2) In case two cycles of sexual activity are called forth, the summer plumage should also develop twice, and the number of moults should correspondingly increase.

3) In case of suppression of the activity of the gonads throughout the year, neither formation of summer plumage, nor moulting should take place at all. The experiments of Benoit ([1,2]) proved that the sexual cycle in ducks is conditioned by light. The methods of the present work are based upon this statement.

Domestic drakes of the Rouen breed, similar as to plumage to the wild duck (*Anas platyrhyncha*) served as material in our experiments. Some of the birds had been castrated. 4 normal and 2 castrated drakes were kept under natural conditions (control), while the others were subjected to a modified influence of light.

The normal correlation of the three form-building processes we are concerned with (sexual cycles, development of seasonal dimorphism in plumage, and moult), as deduced from observations of the control drakes, is shown in the scheme in Fig. 1.

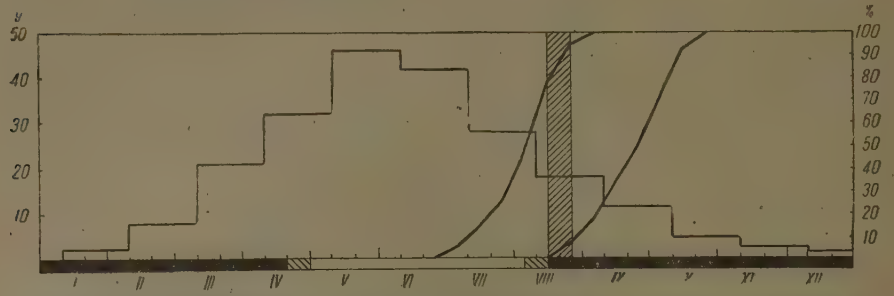

Fig. 1. Cyclic processes under normal conditions. Step-line—change in the weight of the testis. Vertical curves—first and second moult of the plumage on the trunk. Vertical hatched column—moult of the wing. Horizontal column—the black section represents the period of the development of potency for the wedding attire in the feather-building tissue, the white one—for summer attire, the hatched one—mixed potency.

The three normal and two castrated drakes that formed the first series of experiments had been placed from the beginning of the experiment in a room illuminated artificially. On the first day of the experiment, January 20, the

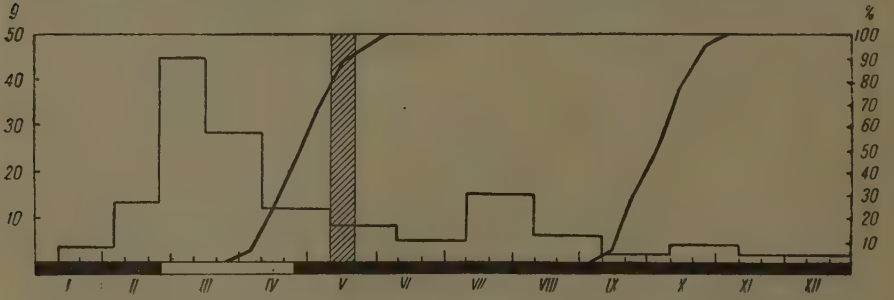

Fig. 2. Untimely activization of cyclic processes. Same denotations as in Fig. 1.

duration of the exposure was that of a normal day (8 hours). It was then increased for 15 min. daily until it reached 17 hours 30 min. (March 15), upon which the exposure was gradually curtailed every day, until reduced to 4 hours.

As at the latitude of Moscow the maximum length of the day (17 hours 34 min.) is reached only on June 23, we aimed to provoke by this series of experiments an untimely sexual cycle.

As may be seen from Fig. 2, we succeeded in shifting the sexual cycle in these birds to February and March, i. e. to a period when the control birds still remained in a state of sexual depression. The duration of the sexual cycle in the experimental birds proved to be considerably shorter than

under normal conditions, according to a more rapid increase or decrease of the exposure (10—15 min. daily, instead of the 5 min. under natural conditions).

It is noteworthy that parallel to this we succeeded in shifting the development of summer attire from April to February, i. e. to a period when under normal conditions the potency for the formation of this type of plumage is lacking altogether.

The period of the decrease of sexual activity was followed by a complete cycle of moulting. Thus, the latter set in three months earlier than it should be under normal conditions. The stimulation caused by light has obviously influenced not only the sexual gland, but other glands too, in particular the thyroid gland, since the change of plumage is conditioned by a definite correlation established between them.

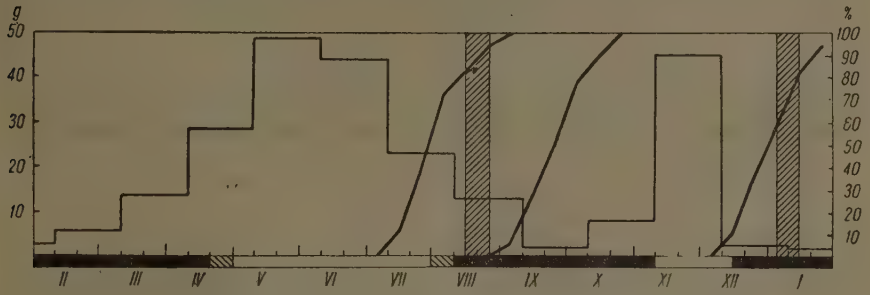

Fig. 3. Recurrence of normal cyclicity. Same denotations as in Fig. 1.

Later on, since the end of July till December, the experimental birds of the first series were kept under normal conditions. As may be seen from Fig. 2, after a short period of depression, we again observed a slight increase in the activity of the gonads; that seemed to be connected with the natural length of the day which is still at its rise by that time. In September, during the second moult, these birds changed their summer (female) plumage to a wedding attire.

The results of this series of experiments fully support the previously made conclusions concerning the dependence of the seasonal changes in the colouring and pattern of the feather in the drake on the activity of the gonads and neither on the season nor on factors localized in the feather-forming tissue itself.

The same is being evidenced by the data of the second series of experiments, involving the induction of the second sexual cycle. Until October these birds remained together with the controls under natural conditions.

As we see from Fig. 3, the form-building processes proceeded in them in the same way as in the control birds. Since October 15, the drakes of this series were placed under the conditions of an artificially lengthened day (which was achieved by adding 5 min. daily for 30 days, and then by curtailing at the same rate until a 8-hour day was obtained). As a result, a second activization of the gonads, the formation of a female attire and a complete change of the plumage took place in November—December.

It should be noted, however, that in spite of a very sharp rise of the curve of sexual activity, all the phenomena connected with the latter fully reappeared.

The data of the experiments described above permit of a general conclusion as to the relations existing between the inner and external factors (light

and sexual activity) that are responsible for the form-building processes (seasonal dimorphism and moult).

This statement is further confirmed by the third series of experiments carried out to investigate the effect of an opposite factor, namely, that of darkness.

When the birds are placed into darkness, the gonads must remain in a state of continuous inactivity, thus, a total absence of form-building processes connected with their activity should be expected.

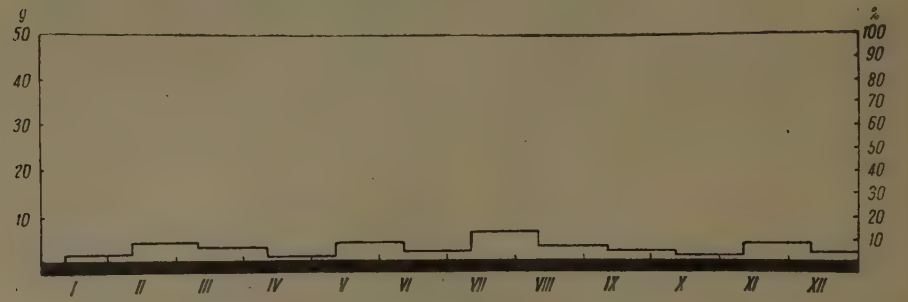

Fig. 4. Suppression of normal cyclicity. Same denotations as in Fig. 1.

The third series consisted of 4 normal and 2 castrated drakes which during the whole year were kept in complete darkness, except for an hour daily, when they were exposed to green light, neutral with respect to the gonads ([3],[4]). This resulted in a total suppression of sexual activity. As may be seen from Fig. 4, the size of the testes remained throughout the year approximately the same, a slight increase being noticed only about the middle

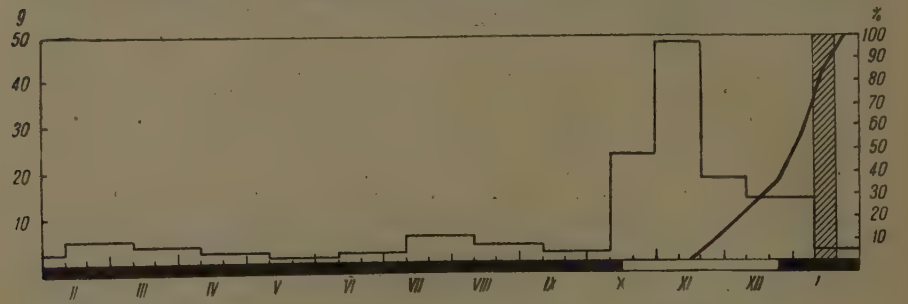

Fig. 5. Shifting of the cycle to a later date. Same denotations as in Fig. 1.

of the year. Under normal conditions, during the period of high sexual activity, the testes attain the weight of 50 g; in our case their maximum weight was 6—8 g. The testes of the drakes kept in darkness had the same appearance as those of the normal birds at the period of sexual depression. It is very important to note that in our case both the potency to the development of summer attire and moulting were depressed. A regular plucking being done every ten days, we always observed a regeneration of the wedding plumage. No signs of moulting either of small or big feathers were observed.

The absence of moult in the birds of the third group should be accounted for by the fact that darkness depresses the function not only of the sexual gland, but of the thyroid gland too. As a result, the function of these glands being simultaneously suppressed, their mutual relations, with regard to the

effect on the feather-forming tissue, prove to be analogous to those existing in normal birds at the period November—February, when the absence of moult is due to reduced sexual activity.

The validity of such an interpretation, as well as the fact that the function of the thyroid gland and its effect upon the moulting also depend on light, are proved by that the castrated drakes went on moulting in darkness, though at a considerably lower rate than those under natural conditions.

In the last, fourth, series the alternating effect of light and darkness was combined. Till October 15 the birds of this series (2 normal drakes) were kept in darkness, after which they were exposed to artificial light, as in the second series. During their confinement in darkness their sexual activity was depressed, seasonal dimorphism and moulting were absent (Fig. 5). Later on, a complete sexual cycle was stimulated by light, potency for the formation of summer plumage having arisen at the peak of the activity of the gonads, while moulting was observed during its decrease.

After our investigation had been brought to end, an article by Walton([8]) appeared in which he reported of having succeeded to obtain premature potency for the formation of summer attire in ducks.

Received
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XX, № 4

ZOOLOGY

ON THE QUESTION OF THE COMPOSITION AND ORIGIN OF THE FAUNA OF THE POLAR BASIN BASSALIA

By E. GURYANOVA

(Communicated by S. A. Zernov, Member of the Academy, 22. V. 1938)

In connection with the intensive studies of the Arctic fauna by Soviet polar expeditions, there has arisen one of the most interesting problems of Arctic zoogeography, namely, the elucidation of the character of the composition and distribution of the deep sea fauna of the Polar basin. These data should serve as a clue to the understanding of the history of development of the Arctic marine fauna, and throw light on the problem of possible division of the Great Ocean bassalia into independent zoogeographical zones, as well as the question about genetic connection between the deep sea faunas of its three principal basins—the Pacific, Atlantic and Arctic. The collections of the First High Latitude Expedition in ice-breaker «Sadko» in 1935 permit of a general characteristic of the abyssal fauna of the Arctic basin.

The conclusions derived herein are based on the analysis of the fauna of higher forms of crustacea (*Isopoda* and *Amphipoda*) of the Arctic and its comparison with the faunas of the Atlantic and Pacific oceans. These two benthonic groups are very convenient for the treatment of zoogeographical problems, as they do not display any planktonic stages whatsoever. Therefore, the bottom relief, the presence of barriers in the form of submarine ridges or isthmuses, or the existence of straits and connections in present and past geological times, constitute one of the most important factors, limiting their distribution at the present time; hence, if one knows the distribution of *Isopoda* and *Amphipoda*, one may judge about modern and ancient connections between sea basins and the history of their origin and development.

The abyss of the Polar basin, as far as its bottom relief is known, falls into three depressions, separated from each other by submarine ridges; 1) depths of 4000 m and over, in the region of the North Pole (the Central depression), enclosed between the continental platform of Eurasia and North America, and isolated from the adjacent depths in the east by the shallow waters of the Chukotsk Sea and in the west by the Fr. Nansen ridge; 2) depths of the Greenland and Norwegian Seas, down to 3300 m (Scandian depression after De Geer), bounded on the north by the Fr. Nansen ridge and on the south by the W. Thomson ridge; 3) depths of 3000 m in the Baffin bay, lying between the shallow waters of the Smith strait in the north and the Holsteinberg Cap Walsingham ridge in the south. :Consequently, the question arises, in the first place, about the uniform composition and origin of the fauna of the Polar basin bassalia. The materials gathered by «Sadko» in 1935 permit to establish definitely the fact, that the abyssal fauna of the Polar basin is essentially uniform, and all its three principal depressions are populated by an identical benthonic fauna. Lists of *Amphipoda* and *Isopoda*, made up by me for depths from 600 to 3300 m for the Scandian, Baffin and Central depressions of the Polar basin, have shown that of a total of

174 species known from those regions 110 species, i. e. 62.6%, prove to be common. The difference in the composition of the faunas of the Central and Scandian depressions is partly due to the rather scant knowledge of the Arctic bassalia, though in the main it depends upon the presence of some allochthonous elements in the depths of the Greenland and Norwegian Seas. The abyssal fauna of the Scandian depression appears to be richer and more diversified than the fauna of the Central basin, as it has an admixture, first, of abyssal Atlantic species, and secondly, of forms of the continental plateau of the North Atlantic, Pacific Ocean and the high Arctic. Thus, in the depths of the Scandian depression thrive Atlantic deep water species, for instance; *Macrostylis subinermis*, *Eurycope brevirostirs*, *Cyphocaris anonyx*, *C. bouvieri*, *Hippomedon longimanus*, *Urothoe abbreviata*, and others, which are not extending any farther, beyond the borders of the Fr. Nansen ridge, into the central region of the Polar basin. The specific hydrological conditions of the Greenland Sea (the sinking of warm Atlantic waters to great depths and sharply pronounced vertical circulation of water masses, intermixing subsurface water layers with bottom layers*) cause in the region of the Scandian depression a curious phenomenon of submergence of more or less shallow water forms, penetrating thither from the continental plateau of the Atlantic and Pacific Ocean, as well as of forms from the Arctic. For example: *Calathura brachiata*, *Desmosoma globiceps*, *Eurycope producta*, *Tryphosa compressa*, *Chironesimus debruyini*, *Pardalisca abyssi*, *P. tenuipes*, *Epimeria loricata*, *Neohela monstrosa*, etc., widespread in the North Atlantic on the continental plateau, after crossing the W. Thomson ridge, descend in the Scandian region to great depths and invade the abyss. Likewise, the relatively shallow water Pacific forms, which occurred widely spread along the Polar coast of North America (e. g. *Arcturus baffini*, *Pleuroprion hystrix*) and many forms of the Arctic continental plateau (for instance, *Munnopsurus giganteus*, *Munnopsis typica*, *Halirages elegans*, *Anonyx nugax*, *Stegocephalus inflatus*, species of the genus *Rhachotropis*, and others) display the same kind of submergence in the Greenland Sea and form part of the abyssal fauna of the Scandian depression. Besides, the abyss of the Scandian depression is inhabited by its own endemic forms from the Arctic autochthons (*Mesidothea megalura*, *Eurycope incisa* n. sp., *Onisimus leucopis*, *O. sextonae*, *Cressa abyssicola*, *Phippsiella minima*, *Lepechinella arctica*, etc.).

The distinction between the abyssal faunas of the Scandian and Central depressions of the Polar basin is due not so much to the difference in the present hydrological conditions, as, before all, to historical causes. The analysis of the main body of the abyssal fauna of the Arctic, consisting of Arctic autochthons, has shown that the fauna of the higher forms of crustacea includes none of the autochthonous Arctic abyssal genera whatever. There are found only autochthonous Arctic abyssal species, belonging to typically shallow water Arctic genera, like, for instance, the abyssal species of shallow water genera *Mesidothea*, *Ilyarachna*, *Onisimus*, *Monoculodes*, *Paroediceros*, *Halirages*, etc. But such peculiar Arctic deep water species, with a limited vertical distribution, are very few in number, while the remaining bulk of the Arctic species, encountered in the depths of the Polar basin, are also met on the continental plateau and are marked by wide bathymetric distribution.

These facts (the absence of peculiar abyssal genera, the invasion of the depths by representatives of shallow water genera and forms of the continental plateau) testify to the relatively young age of the Arctic bassalia. De Geer is of the opinion, that the depths of the Scandian depression origi-

* Fr. Nansen believed the Jan Mayen region to be the centre of the formation of the Polar cold bottom waters.

nated at the transition between the Tertiary and Quaternary periods; it is quite probable that the sinking in the central part of the Polar basin occurred synchronically or even at a later time.

While, as the analysis of the fauna of crustacea of the Continental Arctic plateau (see a series of articles by E. Guryanova on *Isopoda* and *Amphipoda* of the Arctic) has shown, the Polar basin, as a sea basin, has been in existence since ancient times (from the beginning of the Tertiary period, at any rate), and there has evolved an original shallow water autochthonous fauna, represented by a whole series of endemic genera, the same could not be maintained in respect to the Polar basin; evidently, the depths of the Polar basin are comparatively young and have become invaded by representatives of an ancient Arctic shallow water fauna. Those typically abyssal species of isopods and amphipods, which were met in the abyss of the Polar basin, belong either to (1) abyssal Atlantic forms (*Eurythenes gryllus*, *Cyphocaris anonyx*, *C. bouvieri*), or (2) to endemic abyssal species of the Arctic, but pertaining to Atlantic genera (*Lepechinella arctica*, *Cyphocaris polaris* sp. nov)., or else (3) to abyssal species of shallow water Arctic genera (*Mesidothea megalura*, *Onisimus leucopis*, *O. sextonae*, *Paroediceros macrocheir*, *Monoculodes coecus*, *Halirages quadridentatus*, *Eusirus holmi*, and others), or, finally, (4) to forms of the Arctic continental plateau (*Munnopsis typica*, *Munna hanseni*, *Ilyarachna hirticeps*, *Anonyx nugax*, *Hippomedon holbolli*, *Tmetonyx cicada*, *Haploops setosa*, *Harpinia mucronata*, *Lilljeborgia fissicornis*, *Ischyrocerus assimilis*, and many others).

When comparing the Polar abyssal fauna with the abyssal fauna of the Atlantic, inhabiting depths of 3000 to 4000 m southward of W. Thomson ridge, it becomes evident that these faunas are entirely different. Of the total number of species (198 species) of *Isopoda* and *Amphipoda*, known from the abys of the North Atlantic, only 38 forms are found also in the Polar basin; out of that number should be excluded, first, such forms as *Calathura brachiata*, *Janira maculosa*, *Eurycope producta*, *Ambasia atlantica*, *Tryphosa hörring*, *Stegoplax longirostris*, *Pardalisca abyssi*, *Ampelisca latipes*, *Stegocephaloides auratus*, etc., which are marked by a wide vertical range of distribution in the Atlantic, and after crossing the W. Thomson ridge spread in the Polar basin together with the Atlantic waters, along the slope of the continental shelf, without descending any more into the abyss; and, secondly, such species, as *Munna acanthifera*, *Ilyarachna hirticeps*, *Munnopsis typica*, *Stegocephalus inflatus*, *Haploops tubicola*, and others, which being widely spread on the Arctic continent, move through troughs into the North Atlantic, sinking there to depths mostly in the region of the Faeroes canal, the Norwegian trough and Skagerak and Kattegat.

The abyssal fauna of the Polar basin is considerably poorer in species and a great deal more uniform than the Atlantic one. Entire groups, such as, for example, of isopods *Valvifera* and *Flabellifera*, represented in the Atlantic by numerous genera and species (*Cirolana*, *Aega*, *Rocinela*, *Arcturus*, *Astacilla*, *Anthelura*, etc.), either do not occur in the Arctic at all, or else are represented by single specimens. Typically abyssal genera of the Atlantic—*Janirella*, *Dendrotion*, *Pseudomunna*, *Ischnomesus*, *Stortyngura*, *Syneurycope*, *Paramunnopsis*, *Trischizostoma*, *Euonyx*, *Amaryllis*, *Metacyphocaris*, *Paracyphocaris*, *Crybelocephalus*, *Thoriella*, *Koroga*, and others—have no representatives in the Arctic whatever. Genera, which are common both for the bassalia of the Polar basin and the Atlantic, are represented by various species, which are peculiar to one or the other bassalia, e. g. genera *Eurycope*, *Nannoniscus*, *Orchomene*, *Tryphosa*, *Haploops*, *Harpinia*, and others.

All these facts show that we are dealing with two essentially different abyssal faunas and that, consequently, the bassalia of the Polar basin pre-

sent an independent zoogeographical zone. Comparison with the Pacific abyss brings forth a still more striking contrast. The aforesaid is indicative of a very slight interchange of abyssal faunas of the Polar basin and the Great Ocean at the present time. However, the fact that various species, peculiar to each zone, are common both to the Arctic and Atlantic abysses, points to the existence of ancient links between these two basins. Hence, one arrives at the conclusion of non-existence of a continuous land-bridge between Greenland and Europe directly in the preglacial and glacial period.

Exact data on the age of the depths of the Polar basin and on the existence or non-existence of a Greenland bridge in the glacial period may be obtained only by geologists, and in this respect analyses of ground samples, taken by soundings from the depths of the Polar basin by the expeditions «Sądko» in 1935 and by P. Shirshov in 1937—38, will play a decisive part; however, zoogeographical data too authorize one to advance the opinion, namely, first, that the depths of the Polar basin, notwithstanding the antiquity of the basin proper, are relatively young and originated, probably, not before the very end of the Tertiary, and, perhaps, even at the beginning of the Quaternary period, and, secondly, that the Polar basin in the glacial time was not completely isolated from the Atlantic.

If the depths of the Polar basin were of the same ancient origin as the basin itself, i. e. had existed at least from the beginning of the Tertiary period, the Arctic bassalia should have contained autochthonous abyssal Arctic genera, as it is the case on the Arctic continent; yet the existence merely of peculiar abyssal species and the invasion of the polar abyss, in the main, by forms of the continental plateau, speaks for a more recent age of this abyss. On the other hand, the presence of elements of the abyssal Atlantic fauna testifies to a postglacial connection between the Atlantic and the Polar basin, and the existence of peculiar Arctic abyssal species, pertaining to peculiar abyssal Atlantic genera, confirms the connection of these two basins in the glacial period, i. e. the absence of a continuous land-bridge in the place of the present-day W. Thomson ridge. The curious fact of the segregation of deep water species from shallow water genera in the Polar basin, the widely developed capacity of submergence and the existence of a wide bathymetric distribution among the majority of Arctic species of crustacea may possibly be referred to those hydrological changes in the border zones of the Polar basin during the glacial period (cooling and admixture of fresh water) which called forth the migration of fauna into deeper horizons and the adaptation of some of the shallow water forms, driven into the depths, to a deep sea mode of life. It seems to me that exactly in such way occurred the isolation into independent species of purely marine abyssal *Mesidothea megalura*, *Onisimus leucopis*, *O. sextonae*, and others, in the Scandian depression, which having been connected with the Atlantic in the glacial period, had higher salinities than the central part of the Polar basin, where under the influence of a strong admixture of fresh water, a number of species segregated, that were adapted to various degrees of salinity (shallow water species of the genera *Mesidothea*, *Onisimus*, *Pseudolibrotus*, *Monoculodes*, *Oediceros*, *Gammaracanthus*, *Pontoporeia*, etc).

From a brief account of data on abyssal fauna of crustacea of the Polar basin, it is still possible to answer the questions raised in the beginning of this article: first, that the abyssal fauna of the Polar basin is original, autochthonous and of relatively recent age, and secondly, that the Polar basin bassalia should therefore be differentiated into an independent zoogeographical zone, analogous to the Atlantic and Pacific.

Zoological Institute.
Academy of Sciences of the USSR.

Received
10. IV. 1938.

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